SSC-SR-1038

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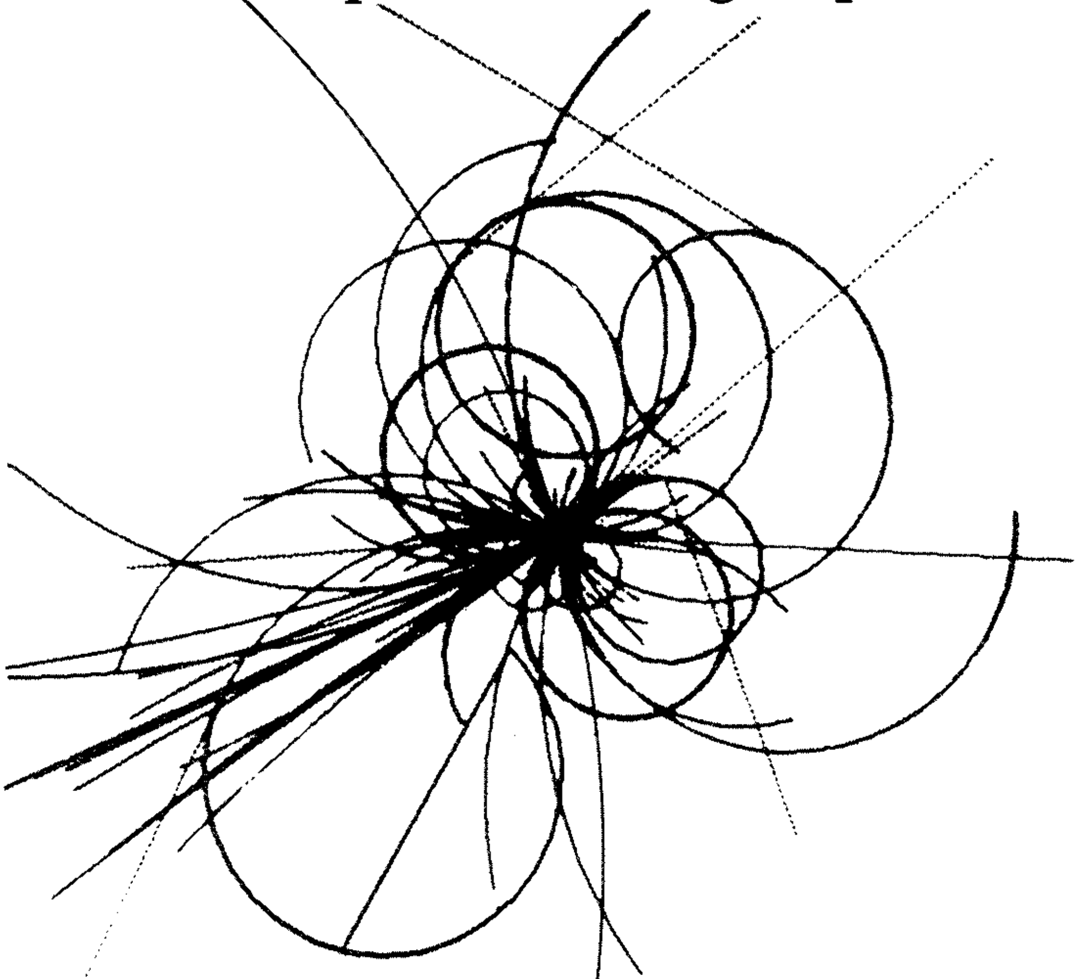

Compensation of SSC Lattice Optics in the Presence of Dipole Field Errors

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SSC Central Design Group

February 1989

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REPORT OF THE CORRECTION ELEMENT WORKING GROUP

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APPENDICES.

CHARGE TO THE WORKING GROUP

The assignment of the Correction Element Working Group (CEWG) is to advance the designs of various candidate correction schemes to a point where they can be compared and distilled down to a single plan. Choosing among the options often involves consideration of incommensurate factors such as cost, practicality, and theoretical performance. Except for minor issues, the CEWG purpose is to gather and array the facts in a form from which these decisions can be rationally made, but not to make the decisions. The present report analyses various schemes for compensating nonlinear multipole errors in the main arc dipoles. Emphasis is on comparing lumped and distributed compensation, on minimizing the total number of correction elements, and on reducing the sensitivity to closed-orbit errors.

1. Introduction and Some Salient Points in the Report.

This report summarizes much of the work performed by the Correction Element Working Group (CEWG) during the last half of 1988. Because the cost of the SSC is dominated by the main arcs, and because the optical defects of those arcs are dominated by field errors in the superconducting dipoles, this study focused on compensating for those field errors, both systematic and random.

The study consisted of many small investigations, some flowing from one to the next and others more independent. The general plan of this report is to describe the former, sequential material, in the body of the report and describe the latter, more independent material, in appendices. In both cases an effort has been made to make the individual sections self-contained; that has led to a certain amount of duplication. Worse, there is a certain amount of inconsistency among different sections, resulting from the large number of variables and from refinements introduced as the study progressed. This makes the report have somewhat of the character of a chronicle with the early sections describing the plan of the study, formulated in advance, and later sections its execution. At no small cost in overall comprehensibility this preserves the motivation and the chronology and permits concentration on specific topics.

The most extreme example of this "historical" approach relates to remote compensation. In Section (6) the great "break-through" of remote correctors for the very satisfactory and economical compensation of systematic errors is described; later in Section (10) it is learned that closed-orbit errors makes remote compensation not necessarily such a great bargain after all. Since remote compensation has been a part of all existing accelerator designs it is important to clarify this, but this history is not yet complete—to complete it is a natural candidate for the continuation of these investigations in the future.

A similar and closely related adventure, described in Appendix E, had to do with the lattice modifications needed to make room for remote compensators. At first it was found that non-uniform cell lengths led to unacceptable deformation of the lattice functions. But the lattice modifications needed to overcome this problems were soon found, as is described in the appendix.

The next two sections, (2) and (3) were written as the study commenced, and describe the plan; it has largely been adhered to. In the present section some of the salient features which have been uncovered will be mentioned. There is however no point in attempting to enumerate all that has been learned as there are far too many details. The only help to be offered here is the mention of our convention of usually choosing units such that the maximum tolerable deviation from the norm, according to the CDR specifications, is 10 units. In interpreting the innumerable tables in this report then, entries are good or bad depending on whether they are small or large compared to 10. That leaves only the problem of figuring out what is being recorded and what are the values of the relevant parameters. Jargon used but not explained in this section is more clearly explained in later sections. Salient points of the report are indicated in the following enumeration, general conclusions are in the concluding chapter.

(1.) Many schemes, lumped and distributed, local and remote, have been found to be capable of compensating systematic multipoles of the expected strengths, provided those errors are the only errors present. The worst errors of this sort are due to persistent currents in the superconducting magnets. These results are mainly contained in Section (5). Remote compensation results are described in Section (6).

(2.) Lumped compensation schemes have been found which are entirely satisfactory; they all share the requirement that at least one corrector in half-cell interiors is required. With the expected systematic field errors it is not possible to control the large amplitude tunes to the required accuracy using only correction elements situated in the spool-piece elements beside the regular arc quadrupoles. Our best efforts at doing this are described in Section (8).

(3.) The same schemes, again both lumped and distributed, that yielded acceptable systematic compensation, have been shown to be capable, within the "needed aperture", of reducing the "smear", present after random errors are included, to acceptable levels of about 5%, after "binned compensation" of just the erect sextupole errors. These result are mainly contained in Section (9).

(4.) Closed orbit errors have been included in the later simulation studies. In theory, the r.m.s. orbit error after correction will be ± 0.4mm. In our study, however, we have regarded it prudent to insist on being able to tolerate errors great enough to lead to an r.m.s. orbit error of ± 1mm. The main deleterious effect of these orbit errors is that the systematic multipoles, carefully compensated up to this point, now act as random multipoles through feed-down effects. At the level of simulating SSC performance in the computer this first makes itself known by causing difficulty in decoupling the lattice. For local compensation schemes, of both the distributed and lumped varieties, this extra difficulty has been judged acceptable. For remote schemes this problem is more serious, and furthermore the smears resulting from this new source of randomness are getting to be marginal. These results are described in Section (10).

Since effects like quadrupole errors and intersection region complications are being intentionally left out, it is only a kind of "best-case" analysis to which the accelerator is being subjected. Of effects studied so far, inclusion of closed-orbit errors has given the most difficulty; for the same reason the observed sensitivity to these errors has given the greatest selectivity among the competing schemes considered. In item (6) below, this sensitivity is quantified to give a "figure of merit", based on this consideration alone, for assessing accelerator parameters. Of course, one can never be sure that problems such as this are not being overrated, owing to insufficient sophistication in the correction procedures, and it is important that they be attacked analytically.

A recommendation following from this is that sensitivity to closed-orbit errors continue to be regarded as an essential criterion in proceeding toward a final correction package. This amounts to just one more step toward the acknowledgement that the inter-relationships are sufficiently complicated that the design cannot be reliably frozen without at the same time simulating the realistic and successful operational use of the elements included in the design. Another illustration of this "operational simulation" philosophy is contained in Section (8); it demonstrates the relative ease of compensating small-amplitude tunes operationally, but the accompanying large amplitude behavior is unacceptable.

(5.) It is important to keep in mind the fact that all statements which are to be made hinge on the actual field errors assumed. Significant deviation of the actual magnet field quality could obviously have important consequences, such as forcing re-design and re-optimization of the correction schemes. Section 4 describes the magnet field error values used in the study. Appendix D(a) contains up-to-date estimates of the field errors. It also contains comparisons between theoretical values and values obtained by measuring early magnets and prototypes. As such these data can be used by the reader in assessing the reliability of the multipole errors which have been used. Of the systematic errors, b_2 and b_4, due to persistent currents present at injection time are the most important. The most important random error is b_2.

(6.) For this study the lattice parameters were mainly held frozen at their CDR values (some exceptions being 90° cells and studying lumped corrector schemes not contemplated in that report). There was no systematic investigation of what could be "bought" by more favorable choices of main parameters like bore size and injection energy. As mentioned above though, the degree of difficulty we found in our narrow investigations can be quantified to give our input to important issues such as that. Two possibilities that can be considered are doubling the injection energy, and increasing the dipole bore diameter, say from 4 cm. to 5 cm. The field multipoles which would accompany those changes are given in Appendix D(b). Some projections as to the improvements which would result follow:

(*i*) Doubling the machine energy would reduce the systematic injection values of b_2 and b_4, the leading offenders, by factors of $3.0/7.4 = 0.40$ and

$0.20/0.64 = 0.31$ respectively. The latter factor could be applied directly, as an improvement factor, to the "worst thing found" in this study, which was mentioned in item (4) above; since the closed-orbit errors would presumably be independent of injection energy, only the absolute error multipole value would enter into the calculation of the feed-down.

(*ii*) The small term b_4 was deemed more important than the large term b_2 in the previous point only because the large b_2 term was assumed to be already compensated. This would still be necessary after doubling the injection energy, though naturally it would be much less critical.

(*iii*) Similar statements about systematic errors could be made about increasing the bore diameter by 25%; the b_2 and b_4 ratios would be $4.7/7.4 = 0.63$ and $0.30/0.64 = 0.47$ respectively. The fact that a 25% increase in bore diameter yields more than a factor of two improvement in this particular aspect of transverse behavior can be ascribed to the unhappily slow convergence of the multipole series.

(*iv*) As regards dynamic aperture, it is the r.m.s. b_2 value which is of greatest importance; the "linear aperture" can be expected to vary inversely with that quantity. Increasing the bore diameter yields an improvement factor of $2.0/1.36 = 1.47$. Doubling the injection energy reduces the "needed aperture" by a factor approaching $\frac{1}{\sqrt{2}} = 0.71$, but actually closer to 0.8 because of the closed-orbit requirement not changing, and possibly not even that good, depending on the assumptions made about injection steering errors.

2. Some General Considerations Regarding Compensation of Lattice Defects.

Much progress has already been made (as noted earlier, this and the next two sections were written as the study commenced in August, 1988) on nonlinear compensation and the schemes still to be analysed already represent a distillation of the original possibilities. Some of the recently completed work is reviewed in Ref. 3; that will not be repeated here, except that known merits and demerits will be included in the verbal description of each scheme.

Degradation due to nonlinear field errors has been quantified mainly by tune shifts and by "smear". In analysing the SSC the following approximate "principles" have repeatedly been found to be valid: (i) systematic bend errors cause tune shifts and not much smear, and (ii) random bend errors cause smear and not much tune shifts. Recent studies (described below) have eroded this separation a bit, since random orbit errors, in combination with systematic magnet errors are found to contribute significantly to smear. For that reason, though we will concentrate primarily on compensating dipole nonlinear field errors, we cannot neglect closed orbit errors, which conspire with other errors to degrade performance. Quadrupole field errors in the dipoles also have a deleterious effect. For this study the main lattice quadrupoles will be treated as perfect.

Ideally the dipole fields would be perfect, and next best would be compensation coils precisely superimposed on the errors they are correcting. Both of these are unrealistic, and the compensation elements will always be somewhat remote from the errors. We will, however, use the term "remote" in a much more exaggerated sense to imply correction elements which are displaced by at least one, and typically many, cells from the error they are compensating. "Local" will mean "in the same half-cell." It is hard to make a similar distinction for the term "lumped compensation". In the past this term has been restricted to coils of essentially zero length, and has been contrasted to "bore-tube" correctors which run an appreciable fraction of the length of the dipole they are part of. In studies already completed, and discussed further in Appendix C, it has been found that bore-tube coils are dynamically lumpy unless they are full length or unless they are centered in the dipole. Also, an option introduced as possibly leading to manufacturing simplification has short correctors built on the bore-tube at the dipole ends. For these reasons the adjectives "lumped" and "bore-tube" are no longer mutually exclusive.

There are two attitudes which can be taken toward error compensation, one more conservative than the other. The conservative approach could be called "dead-reckoning" or "simple-minded" —this is not pejorative; it means the intentional choice not to be committed to the use of information which may not be available—and it assumes that correctors settings are based purely on measured errors in the individual components, with no dependence on accelerator beam measurements. For small, and especially for historically-early accelerators this was the natural approach. The intention, not necessarily successful, was to assure first-time operation. The other approach, which assumes that beam measurements will be used for setting correctors, can be called "operational." The best example of the historical evolution from dead-reckoning to operational compensation is closed-orbit flattening. Early, small accelerators were constructed with an accuracy assuring an acceptable closed orbit upon turn-on; but for modern large rings it is impractical to hold tight enough tolerances to keep the closed orbit in the machine. For the SSC other operations, such as chromaticity control and decoupling, will also require operational compensation, both because tolerances cannot be held to small enough values to assure successful operation without such compensation, and because some measurements on which "dead-reckoning" is based will be imprecise enough as to necessitate further correction using beam measurements (90% of the dipoles will be measured only warm, so that if the cold-warm correlation is erratic, "dead reckoning" may be inadequate). For planning purposes the operational approach is studied by "operational simulation"; description of such studies forms part of this report.

3. General Strategy for Zeroing in on the "Best" Design or Designs.

The correction issue impacting most strongly on the SSC plan is whether to use bore-tube correctors (as in the CDR) or lumped correctors. To be more precise, since some lumped correctors will most likely be required, the two possibilities are: no bore-tube correctors, or bore-tube correctors for the largest multipole or multipoles. Several candidates of each of these types will be studied. As far as lumped schemes are concerned, it is known to be necessary,[4–6] for the dominant multipole, to have at least two correctors in at least some half-cells, with one being in the interior of the half-cell (i.e., not adjacent to the quadrupole). It is also known, for compensation of purely systematic errors, that more than two correctors per half cell is unnecessary.[4,5] We restrict our investigations to only the two most promising lumped configurations; they are called the "Simpson-Neuffer" scheme and the "Gaussian Quadrature" scheme. The theoretical basis for considering these two schemes to be desirable is included in Appendix A, and a more detailed description is given below.

Three forms of theoretical (numerical) analysis are employed, each starting with a prescription for setting the correctors, continuing by tracking extreme particles for some hundreds of turns, and finishing by extracting the smear and tune for each particle. They are:

(*I*) Compensation of systematic magnet multipoles assuming no random magnet errors and no closed orbit errors.

(*II*) Compensation of random magnet errors in the presence of already corrected systematic errors but with no closed-orbit errors.

(*III*) Inclusion of random closed orbit errors and other errors.

Since analysis (II) is much more computer-intensive than analysis (I), it was sensible to subject a much broader class of candidates to (I) than to (II). Only the best performers were retained, and they fully met the CDR specifications, which will be spelled out in the next section. Similar comments apply to analyses (II) and (III). Analysis (III) is very computer-bound (and also brainpower-bound) as it amounts to a full operational simulation of orbit control, tune control, and chromaticity control. Only schemes which were fully satisfactory at the level (II) analysis were subjected to this analysis. The final schemes meet the requirement of constancy of the off-momentum, large-amplitude tunes, are sufficiently fine-grained to yield satisfactory improvement of the linear aperture by means of the "binning" compensation of random errors,[7] and have been checked to be satisfactory for chromaticity adjustment and orbit flattening. Sensitivity to errors which are partly random, partly systematic, has not been studied.

With the study being organized as has just been described, and assuming that there is a satisfactory level of understanding at each stage, any theoretical basis for favoring one scheme over another amounts to stating which performs better under analysis (III); more specifically, which is more tolerant to closed orbit errors after the systematic and random multipole errors are properly compensated for according to prescription? As mentioned earlier, this is just one of the factors, along with other considerations like cost, ease of manufacturing, and practicality, which will go into deciding among them.

Theoretical calculations have been described in References 6. Particle tracking has been performed using TEAPOT. Various aspects of operational simulation have been described in References 8-10. In Ref. 11 it is shown that breaking each dipole in half, with each half treated as thin, yielded tunes accurate to ± 0.0001 (with respect to analytical calculations), and that is what has been done for this report.

4. Assumed Errors and Specification of Performance.

The expected field errors are given in Appendix D(a), which updates estimates contained in the CDR based on recent experience. The main systematic errors are $b_2 = -7.4, b_4 = 0.64$, and $b_6 = -0.13$ coming from persistent currents at the injection energy. These are in the usual units of parts per 10^4 at one centimeter. They are larger than the saturation multipoles though the latter may demand more powerful correctors since they appear at full field. The main random errors have standard deviations given by $\sigma_{b2} = 2.0, \sigma_{a2} = 0.6, \sigma_{b3} = 0.3$, and $\sigma_{b4} = 0.7$.

A lattice consisting only of 320 simple 90^o FODO cells, with parameters identical to those in the regular arcs of the SSC, is assumed. In all cases the tunes were adjusted to the values $Q_x = 81.285, Q_y = 82.265$.

The degradation of accelerator performance due to field errors will, as usual, be discussed by quantifying the tune variation and the smear. The independent variables, (x, y, δ), are the horizontal and vertical transverse amplitudes and the momentum offset. For this report calculations are performed with these variables set to the following extremes:

(i) Near the origin, for reference.

(ii) With $\delta = 0$: $x = 5$mm at the point in the lattice where β_x is maximum and $y = 5$mm at the point in the lattice where β_y is maximum; this is the point along the diagonal in (x, y) space at which analyses have conventionally been performed; call it $(5, 5)$. Points $(5, 0)$ and $(0, 5)$ were also studied to guard against the possibility of accidentally good behavior on the diagonal.

(iii) With $\delta = \pm 0.001$: (3.6,3.6), (3.6,0.0), and (0.0,3.6). For some previous investigations[6] in which effects of individual multipoles were calculated independently, the on-momentum and the off-momentum specification amplitudes were increased from 5 and 3.6 mm to 7 and 5 mm respectively. This conservatism was intended to allow for possible undesirable conspiracy between different multipoles, whose effects were being calculated individually. Some results with those larger amplitudes will also be shown.

In subsequent tables, values of the tune difference

$$\Delta Q_x(x,y,\delta) = Q_x(x,y,\delta) - Q_x(0,0,0) \qquad (4.1)$$

at these extreme points will be presented, in units of 0.001. and $\Delta Q_y(x,y,\delta)$ will be exhibited similarly. Smear values in percent, $S(x,y,\delta)$, will also be given.

For the SSC the following performance specifications have been set for values of the variables in the interior of the region defined by the above extremes:

(i) The maximum tune variation should remain in the range ± 0.005.

(ii) The smear should remain less than 10%. This specification is not entirely uncontroversial and is subject to continuing study, for example experimentally in the Tevatron experiment E778. Even the definition of smear is not universally established in cases where the x and y invariant amplitudes are unequal. In this paper the smear is taken to be the bigger of an x-smear and a y-smear defined as follows. The normalizing amplitude, $\bar{a}$, for both is taken to be $\sqrt{a_x^2 + a_y^2}$ where $a_x = \sqrt{x^2/\beta_x}$ and $a_y = \sqrt{y^2/\beta_y}$ are invariant amplitudes averaged over the motion. The x-smear is $\sqrt{2}/3 \times (a_x^{max} - a_x^{min})/\bar{a}$ and the y-smear is $\sqrt{2}/3 \times (a_y^{max} - a_y^{min})/\bar{a}$. The off-momentum smear is calculated using the same formulas except that the correct off-momentum lattice functions are used and the transverse amplitudes are measured relative to the appropriate off-momentum closed orbit.

In the tables of both tune variation and smear the units have been chosen such that 10 marks the boundary between acceptable and unacceptable performance according to these specifications.

5. Comparison of Various Schemes on the Basis of Tune Variation Due to Systematic Errors.

In this section it is assumed that the only errors present are systematic: $b_2 = -7.4$, $b_3 = 0.1$, $b_4 = 0.64$. These errors are large enough, if uncompensated, to give unacceptably large smear, but after compensation by any of the following schemes the smear is less than 1 percent and will not be exhibited. Before introducing any errors the tunes were set to their nominal values and both chromaticities were adjusted to zero, using the sextupoles situated next to the main arc quadrupoles (these will be called chromaticity sextupoles).

It is assumed that all of the systematic multipoles are compensated locally, and that there is no "remote" compensation: that is, every cell is treated identically. In a later section it will be shown that, at least in the absence of orbit errors, this is unnecessarily extravagant and can be relaxed so that b_3 and higher multipoles are compensated remotely.

5.1 Small-Amplitude Chromatic Behavior of Various Bore-Tube Configurations After Simple-Minded Compensation.

The data in this section are intended to illustrate the delicacy of the required chromatic compensation. In the current SSC design the b_2 bore-tube correctors run from one end of the dipole to about the middle. Magnets like that could be installed in the ring in various orders and orientations.

$$\Big|\Big)\ [c-][c-][c-]\vdots[c-][c-][c-]\ \Big)\Big(\ [c-][c-][c-]\vdots[c-][c-][c-]\ \Big(\Big|\text{BORASYM}$$

$$\Big|\Big)\ [c-][c-][c-]\vdots[c-][c-][c-]\ \Big)\Big(\ [-c][-c][-c]\vdots[-c][-c][-c]\ \Big(\Big|\text{BORASYMALT}$$

$$\Big|\Big)\ [c-][c-][c-]\vdots[-c][-c][-c]\ \Big)\Big(\ [c-][c-][c-]\vdots[-c][-c][-c]\ \Big(\Big|\text{BORSYM}$$

$$\Big|\Big)\ [c-][c-][c-]\vdots[-c][-c][-c]\ \Big)\Big(\ [-c][-c][-c]\vdots[c-][c-][c-]\ \Big(\Big|\text{BORSYMALT}$$

The most obvious configurations are indicated in the above cell schematics, in which the symbol [$c-$] stands for a dipole having its b_2 correction coil superimposed on the left half. Each of these correctors was adjusted to make $\int b_2 dl$ over its own dipole vanish. These schemes differ in respect to their symmetries about half-cell mid-points, indicated by dots in the figure. They are symmetric (BORSYM and BORSYMALT) or anti-symmetric (BORASYM and BORASYMALT). In Appendix C there is a discussion of the effect of such symmetry and also an estimate of the importance of small displacements between field error and its compensation. The alternating schemes, BORSYMALT and BORASYMALT preserve the symmetry within half-cells, but magnets in successive half-cells are reversed. The small-amplitude tunes of all these lattices are given in the following Table.

SMALL-AMPLITUDE TUNES						
δ		-0.001		0.0	0.001	
		Q	ΔQ	Q	Q	ΔQ
BORASYM	Q_x	81.2691	-15.9	81.2850	81.3012	16.2
	Q_y	82.2667	1.7	82.2650	82.2634	-1.6
BORASYMALT	Q_x	81.5835	298.5	81.2850	81.0347	-250.3
	Q_y	82.3711	106.0	82.2650	82.1557	-109.3
BORSYM	Q_x	81.4308	145.8	81.2850	81.1437	-141.3
	Q_y	82.2530	-12.0	82.2650	82.2779	12.9
BORSYMALT	Q_x	81.2802	-4.8	81.2850	81.2906	5.6
	Q_y	82.2670	2.0	82.2650	82.2634	-1.6

Remembering that the performance specification is that the largest and smallest ΔQ entry must not differ by more than 10, since the ΔQ entries are in units of 0.001, it can be seen that only BORSYMALT meets the specification; (actually, not quite, since 5.6+4.8=10.4.) This unsatisfactory behavior is explained in Appendix C.

5.2 Small-Amplitude Behavior of Various Bore-Tube Configurations After Operational Chromaticity Compensation.

Next, and for all other investigations in this report, the operational approach was taken, of adjusting the chromaticities to zero, with the chromaticity sextupoles, after the compensators had been set. The results are shown in the following Table.

SMALL-AMPLITUDE TUNES						
δ		-0.001		0.0	0.001	
		Q	ΔQ	Q	Q	ΔQ
BORASYM	Q_x	81.2856	0.6	81.2850	81.2847	-0.3
	Q_y	82.2651	0.1	82.2649	82.2650	0.0
BORASYMALT	Q_x	81.2825	-2.5	81.2850	81.2919	6.9
	Q_y	82.2655	0.5	82.2649	82.2649	-0.1
BORSYM	Q_x	81.2833	-1.7	81.2850	81.2900	5.0
	Q_y	82.2652	0.2	82.2649	82.2653	0.3
BORSYMALT	Q_x	81.2858	0.8	81.2850	81.2851	0.1
	Q_y	82.2651	0.1	82.2649	82.2652	0.2

All performance is much improved.

5.3 Large-Amplitude Behavior of the Same Bore-Tube Configurations After Operational Chromaticity Compensation.

The large-amplitude behavior of the tunes for the same bore-tube compensation schemes is shown in the following figure. The entries are tune discrepancies from the nominal (in units of 0.001) at those standard phase space points defined in Section (4).

LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
BORASYM	ΔQ_x	0.52*	0.51	0.03	0.11	0.09	-0.04	-0.45	-0.46	-0.05
	ΔQ_y	-0.06	0.19	0.20	-0.06	0.12	0.14	-0.06	-0.11	-0.10
BORASYMALT	ΔQ_x	-4.26	-3.29	-1.05	1.52	1.55	0.00	5.84*	4.83	-1.03
	ΔQ_y	0.96	2.98	2.00	-0.03	-1.00	-1.00	-0.90	-4.04	-3.04
BORSYM	ΔQ_x	-3.16	-3.14	0.07	1.05	1.54	0.58	4.13	4.65*	4.26
	ΔQ_y	-0.01	-0.35	-0.39	0.49	2.38	1.82	4.92	2.76	2.22
BORSYMALT	ΔQ_x	0.44	0.54	0.15	0.35	0.56	0.19	-0.19	-0.10	0.09
	ΔQ_y	0.07	0.36	0.24	0.14	0.79*	0.58	0.03	0.40	0.31

In each case the largest entry is marked by an asterisk. Again BORASYM and BORSYMALT are fully satisfactory. Of these, BORASYM, which is the present CDR plan and will, for that reason, from now on be called BORCDR, is noticeably easier to build, since all magnets are identical. Nevertheless, since symmetry about half-cell centers is theoretically attractive, the alternating symmetric arrangement (BORSYMALT), which will be called BORTUB from now on was also retained for subsequent, more demanding, studies.

5.4 Comparison of Small Amplitude Behavior of Various Corrector Configurations.

We next introduced three other candidate schemes, all expected to have excellent properties. They are illustrated below, along with the two schemes surviving the previous analysis. The same cell length was used for all cases (even though it doesn't look that way in the figure).

$$
\begin{array}{rl}
\Big|\Big)\ [c-][c-][c-][c-][c-][c-]\ \Big)\Big(\ [c-][c-][c-][c-][c-][c-]\ \Big(\Big| & \text{BORCDR} \\
 & (\equiv \text{BORASYM}) \\
\Big|\Big)\ [c-][c-][c-][-c][-c][-c]\ \Big)\Big(\ [-c][-c][-c][c-][c-][c-]\ \Big(\Big| & \text{BORTUB} \\
 & (\equiv \text{BORSYMALT}) \\
\Big|\Big)\ c[\ \ ]c[\ \ ]c[\ \ ][\ \ ]c[\ \ ]c[\ \ ]c\ \Big)\Big(\ [\ \ ]c[\ \ ]c[\ \ ]cc[\ \ ]c[\ \ ]c[\ \ ]\ \Big(\Big| & \text{MAGEND} \\
\Big|\Big)\ F[\ \ ][\ \ ][\ \ ]C[\ \ ][\ \ ][\ \ ]\ \Big)\Big(\ D[\ \ ][\ \ ][\ \ ]C[\ \ ][\ \ ][\ \ ]\ \Big(\Big| & \text{SNEUFF} \\
\Big|\Big)\ [\ \ ]G[\ \ ][\ \ ][\ \ ]G[\ \ ]\ \Big)\Big(\ [\ \ ]G[\ \ ][\ \ ][\ \ ]G[\ \ ]\ \Big(\Big| & \text{GAUINT}
\end{array}
$$

The arrangement labelled MAGEND was first suggested by John Peoples and John Rees,[12] motivated by the desire to simplify the dipole manufacturing program by concentrating the bore-tube compensation in a single element at one end of every dipole. They conjectured that the resulting compensation would be to all intents and purposes local, and almost equivalent to the longer, CDR, elements. Based on prejudices described above and in Appendix C, the symmetric arrangement, alternating every half-cell, is expected to perform best, and that is the arrangement which has been studied. For analysis purposes these are lumped correctors, but in the dipole manufacturing and measuring phase they would be the responsibility of the dipole manufacturing group; in that sense they more nearly resemble the bore-tube correctors.

All lumped schemes which have been studied are variants of those labelled SNEUFF (an abbreviation for Simpson-Neuffer) and GAUINT (an abbreviation for Gaussian integration). The rationale behind the lumped schemes is explained in Appendix B. The SNEUFF geometry has the advantage of placing correctors at the center of the cells as well as close to the F and D quadrupoles, which permits separate control of horizontal, coupled, and vertical motions. This allows correction and control of the motion beyond the first-order integration rule cancellations used in this study. These advantages of separate controls have not been explored here. There are theoretical grounds (pointed out by Forest and Neuffer[13]) for expecting GAUINT to be more effective for some nonlinear compensation effects. At the same time that plan, which requires that there be either 5 or 10 dipoles per half-cell, represents the greatest departure from the CDR of all plans studied.

SMALL-AMPLITUDE TUNES						
δ		-0.001		0.0	0.001	
		Q	ΔQ	Q	Q	ΔQ
BORCDR	Q_x	81.2856	0.6	81.2850	81.2847	-0.3
	Q_y	82.2651	0.1	82.2649	82.2650	0.0
BORTUB	Q_x	81.2858	0.8	81.2850	81.2851	0.1
	Q_y	82.2651	0.1	82.2650	82.2652	0.2
MAGEND	Q_x	81.2869	1.9	81.2850	81.2859	0.9
	Q_y	82.2655	0.5	82.2650	82.2655	0.5
SNEUFF	Q_x	81.2911	6.1	81.2850	81.2894	4.4
	Q_y	82.2664	1.4	82.2850	82.2668	1.8
GAUINT	Q_x	81.2899	4.9	81.2850	81.2902	5.2
	Q_y	82.2662	1.2	82.2650	82.2661	1.1

All are satisfactory.

5.5 Comparison of Large-Amplitude Behavior of The Same Corrector Configurations.

LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
BORCDR	ΔQ_x	0.52*	0.51	0.03	0.11	0.09	-0.04	-0.45	-0.46	-0.05
	ΔQ_y	-0.06	0.19	0.20	-0.06	0.12	0.14	-0.06	-0.11	-0.10
BORTUB	ΔQ_x	0.40	0.51	0.09	-0.21	-0.14	-0.04	0.34	0.55	0.19
	ΔQ_y	0.06	0.34	0.23	0.04	0.41	0.33	0.13	0.79*	0.59
MAGEND	ΔQ_x	0.97	1.28	0.37	1.04	1.78	0.85	0.01	0.37	0.40
	ΔQ_y	0.37	1.27	0.98	0.78	1.83*	1.14	0.44	0.58	0.13
SNEUFF	ΔQ_x	2.19	2.84	0.56	3.46	5.65	2.19	0.85	2.40	1.62
	ΔQ_y	0.60	2.37	1.74	2.00	5.77*	3.67	1.46	3.44	1.95
GAUINT	ΔQ_x	1.35	2.30	0.98	3.34	4.74*	1.28	1.68	1.91	0.23
	ΔQ_y	0.91	2.57	1.67	1.28	4.03	2.82	0.31	1.44	1.13

Again, the largest entry for each scheme is indicated by an asterisk. If one were to order them, from best to worst, the ordering would be: BORCDR and BORTUB, followed by MAGEND, followed by GAUINT and SNEUFF. There are two reasons why this is probably inappropriate. For one thing, all schemes meet the specifications, and it is "apple-polishing" to attempt to do better. For another, tiny residual shifts like this will inevitably be present from other sources; as we shall see, they can be reduced as part of the global operational chromaticity compensation.

One should say then, that all schemes are satisfactory at this level.

6. Remote Compensation Results.

6.1 Cases Studied.

For compensation of small multipole errors, remote compensation is potentially economical and satisfactory. The following study was predicated on the hope that all of b_3, b_4, and b_6 could be compensated remotely, where remotely means every 10 or so cells. The following table shows the configurations which have been investigated, including, for reference, the local schemes already introduced.

REMOTE AND LOCAL SCHEMES				
local	remote period			
	6	9	10	14
BORCDR				
BORTUB				
	BORNEU06	BORNEU09	BORNEU10	BORNEU14
		BORGAU09		
MAGEND			MAGENDR	
SNEUFF			SNEUFFR	
GAUINT			GAUINTR	

For the lumped schemes, MAGEND, SNEUFF, and GAUINT, it was natural to use the same lumped locations in the remote cells as in the local cells; b_2 was corrected in every cell, b_3, b_4, and b_6 every 10th cell. The resultant lattices were called MAGENDR, SNEUFFR, and GAUINTR.

The case of retaining b_2 bore-tube correctors was more complicated. First of all a new b_2 local bore-tube correction possibility was introduced; it assumes a full-length b_2 coil in every dipole and is sometimes symbolized by BORFUL. Theoretically, if b_2 is measured perfectly and the full-length coil is set perfectly, it is equivalent to the magnet having no b_2 multipole. For present purposes, that is exactly what is assumed. All the remote schemes beginning with "BOR" in this section therefore are actually BORFUL, having full-length b_2 bore-tube correctors. For the higher multipoles the systematic errors in BORFUL are the same as in all the other schemes. There was no point in investigating local compensation of BORFUL, both because it is necessarily superior to BORCDR, as regards b_2, and because such magnets would not, in fact, have bore-tube correctors for higher multipoles, since there is not enough room for them. It was assumed that lumped correctors were present every 6 cells (BORNEU06), every 10 cells (BORFULR=BORNEU10), or every 14 cells (BORNEU14); different periodicities were studied in order to uncover possible resonant effects. It later was decided that every 9 cells gave good performance analytically and computationally, and was also included in our studies.

To the lowest order in multipole strengths the superposition principle for systematic errors holds, in which a corrector of strength 1, every cell, is entirely equivalent to an identically-placed corrector of strength N, every N cells. For this reason, the same lumped arrangements known to be good locally should be good remotely for sufficiently weak multipole errors. The single most important application of this principle is that it is known that there must be at least one lumped corrector in a half-cell interior if b_2 is to be corrected entirely with lumped correctors. (The issue of whether higher order lumped multipole correctors can be confined to quad location, is the subject of a later section.) For the remote BORFUL possibilities it was decided to use initially a variant of the Simpson-Neuffer lumped arrangement for the b_3, b_4, and b_6 remote, lumped coils. The variation was to retain the F_-, F_+ and D_-, D_+ coils separate, rather than lumping them together into F and D as in SNEUFF (see Appendix B for notation.) On theoretical grounds, this arrangement has to perform at least as well for systematic compensation, plus, being symmetric about the half-cell centers, it may be much less sensitive to other errors. Having three lumped correctors per half-cell, this plan might be regarded as too extravagant, but consideration of that issue will be deferred until after the main issues of remote compensation have been clarified. These remote schemes were labelled BORNEU to indicate that the remote configuration is Simpson-Neuffer. In a later section a remote Gaussian case will be called BORGAU.

Another, more important issue of practical machine design is also being deferred until remote compensation has been demonstrated to be fully effective. The economy of remote compensation is the reduction in the number of correctors by a factor equal to the periodicity N. The saving is partly due to the reduced number of elements and partly due to the reduced space requirement along the ring circumference. Though later found to be not so important, it was originally thought to be highly desirable that, as regards cell-length, steering, and linear optics, the lattice not exhibit the periodicity N; i.e. we do not want every N'th cell to be longer. The importance of this requirement is discussed in Appendix E. If empty drift spaces are inserted in every cell to achieve uniform length, then a large part of the economy of remote compensation is lost. Now, invoking the same superposition principle, there is no reason why all the remote correctors should be in the same cells; i.e., they can be staggered so that the same drift spaces in different cells are used for different multipoles. For simplicity we decided to defer study of this issue by assigning zero length to the correctors and putting them all in the same remote cells.

6.2 Comparison of Various Configurations and Dependence on Remote Corrector Period.

The results of all these investigations are shown in the following tables which are much like the earlier tables in this report.

REMOTE COMPENSATION SMALL-AMPLITUDE TUNES				
δ		-0.001	0.0	0.001
MAGENDR	ΔQ_x	6.5	0	14.7
	ΔQ_y	-0.6	0	2.8
SNEUFFR	ΔQ_x	5.9	0	4.1
	ΔQ_y	1.4	0	1.7
GAUINTR	ΔQ_x	4.8	0	5.0
	ΔQ_y	1.2	0	1.0
BORNEU06	ΔQ_x	1.4	0	-1.7
	ΔQ_y	-0.3	0	0.2
BORFULR=	ΔQ_x	0.8	0	-1.4
BORNEU10	ΔQ_y	-0.1	0	0.2
BORNEU14	ΔQ_x	0.8	0	-0.4
	ΔQ_y	0.0	0	-0.6

All are satisfactory except MAGENDR. (It is not currently understood why that scheme is noticeably inferior in this regard; without looking into this further the above table should probably not be regarded as grounds for rejecting MAGENDR.)

REMOTE COMPENSATION LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
MAGENDR	ΔQ_x	9.60	6.70	-3.30	-1.70	-0.07	2.10	-11.6*	-7.60	4.40
	ΔQ_y	-2.70	-1.50	1.10	1.60	2.80	0.81	-0.06	-0.11	-0.10
SNEUFFR	ΔQ_x	1.30	2.20	0.77	3.40	5.60	0.24	-0.73	1.30	1.90
	ΔQ_y	0.62	2.40	1.70	2.00	5.90*	3.50	-0.90	-4.04	-3.04
GAUINTR	ΔQ_x	1.30	2.50	1.10	3.30	4.80*	1.50	0.90	1.50	0.50
	ΔQ_y	0.97	2.70	1.60	1.30	4.20	2.70	0.40	2.00	1.20
BORNEUR	ΔQ_x	0.44	0.21	-0.23	-0.17	-0.03	0.28	-1.40*	-0.79	0.70
	ΔQ_y	-0.29	-0.28	-0.13	-0.06	0.07	-0.25	0.51	0.85	-0.09

REMOTE COMPENSATION LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
BORNEU06	ΔQ_x	3.40	0.34	-1.10	-0.01	-0.08	0.07	-4.10	-0.96	1.30
	ΔQ_y	-2.90	-1.10	0.05	-0.05	0.04	-0.17	0.51	0.85	-0.09
BORNEU10	ΔQ_x	0.44	0.21	-0.23	-0.17	-0.03	0.28	-1.40	-0.79	0.70
	ΔQ_y	-0.29	-0.28	-0.13	-0.06	0.07	-0.25	0.51	0.85	-0.09
BORNEU14	ΔQ_x	2.40	0.52	0.11	0.29	-0.17	0.14	-3.2	-0.87	0.09
	ΔQ_y	-1.80	-0.67	-0.47	-0.35	0.21	0.11	1.80	1.00	0.70

The conclusion to be drawn from this data is that all remote compensation schemes are satisfactory, with the possible exception of MAGENDR. The performance with $N = 10$ is best, but there is no striking dependence on periodicity of the remote compensation.

Finally, and most importantly for practical machine design, the issue of realistic corrector designs and strengths is discussed in Section 11.

7. Investigation of Very Large Amplitudes to Protect Against Ignorance of Multipole Signs.

7.1 Cases Studied.

CASES STUDIED AT LARGER AMPLITUDES			
	remote period		
	9	32	320
BORCDR			
BORCDRB3			
BORTUB			
	BORNEU09	BORNEU32	BORNE320
	BORGAU09		
MAGEND			
SNEUFF			
GAUINT			

The previous table shows the large-amplitude cases studied.

BORCDR and BORCDRB3 differ only in the sign of b_3. BORNEU09 and BORGAU09 are intended to compare remote Simpson-Neuffer and remote Gaussian. Though periodicity 10 was shown to be perfectly satisfactory above, there is a small logistic advantage (having to do with the number of cells per sector) in using 9, and it is also theoretically favored (see Appendix F). This was checked with BORNEU09, and extreme remote compensation was studied with BORNEU32 and BORNEU320, the latter of which has only one lumped corrector cell in the entire machine. Except for being at larger amplitude, these runs extend the investigation of the dependence on the remote period.

As mentioned before, some analytical investigations of systematic behavior have been performed with the on-momentum and the off-momentum specification amplitudes increased from 5 and 3.6 mm as used in the last section to 7 and 5 mm respectively. This conservatism was originally intended to allow for possible undesirable conspiracy between different multipoles, whose effects were being calculated individually. The signs of some of the systematic multipoles are unpredictable; by calculating with both signs this effect is studied. Then various cases are repeated at the larger amplitudes. Finally, some other remote dependences are studied: dependence on remote period, and a comparison of Gaussian versus Simpson-Neuffer.

7.2 The Effect of Reversing the Sign of b_3.

The signs of the largest systematic multipoles, b_2 and b_4, are predictable since they are due to persistent currents. That means that the leading polarity uncertainty is the sign of b_3. In the next table results are given in two cases, differing only in the sign of b_3.

DEPENDENCE ON SIGN OF b_3 LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		5.0,0	5.0,5.0	0,5.0	7.0,0	7.0,7.0	0,7.0	5.0,0	5.0,5.0	0,5.0
BORCDR	ΔQ_x	0.86	0.93	0.07	0.33	0.38	0.01	-0.56	-0.58	-0.03
($b_3 = -0.1$)	ΔQ_y	0.00	0.49	0.44	-0.01	0.26	0.23	-0.05	-0.20	-0.20
BORCDRB3	ΔQ_x	1.12	1.22	0.07	0.74	0.81	0.03	-0.40	-0.41	0.01
($b_3 = +0.1$)	ΔQ_y	0.01	0.75	0.67	0.01	0.74	0.67	-0.04	-0.01	-0.01

Since the biggest discrepancy is 0.6 units, this uncertainty in the sign of b_3 can be considered to be unimportant. Of course that is largely because its absolute value is as small as it is.

7.3 Local Compensation: Very-Large Amplitude.

The cases studied in the Table in Section 5.5 are repeated here.

LOCAL CORRECTORS VERY-LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		5.0,0	5.0,5.0	0,5.0	7.0,0	7.0,7.0	0,7.0	5.0,0	5.0,5.0	0,5.0
BORCDR	ΔQ_x	0.86	0.93	0.07	0.33	0.38	0.01	-0.56	-0.58	-0.03
	ΔQ_y	0.00	0.49	0.44	-0.01	0.26	0.23	-0.05	-0.20	-0.20
BORTUB	ΔQ_x	0.77	1.02	0.25	0.74	1.12	0.41	-0.04	0.17	0.21
	ΔQ_y	0.20	0.66	0.41	0.35	1.41	1.01	0.14	0.84	0.64
MAGEND	ΔQ_x	3.48	3.78	0.10	1.57	3.30	1.74	-1.93	-0.51	1.60
	ΔQ_y	0.13	2.23	2.07	1.65	3.86	2.21	1.45	1.56	0.11
SNEUFF	ΔQ_x	3.71	4.84	1.08	6.74	10.9	4.30	2.43	5.46	3.07
	ΔQ_y	1.08	4.51	3.57	3.92	11.6	7.49	2.89	6.94	3.86
GAUINT	ΔQ_x	3.02	5.02	2.03	6.51	8.94	2.66	2.87	3.38	0.47
	ΔQ_y	1.91	5.22	3.26	2.36	8.18	5.69	0.50	2.77	2.38

Even at this extreme amplitude all schemes pass, or just barely fail. As an aside, it can be noted that because the important deviations are all positive, they could in principle be much reduced with octupole correctors.[14] At least one of these would have to be at a half-cell center, but they could most likely be quite remote.

7.4 Dependence on Periodicity of Remote Correctors: Very-Large Amplitude.

LOCAL CORRECTORS VERY-LARGE-AMPLITUDE TUNES										
δ		-0.001			0.0			0.001		
		5.0,0	5.0,5.0	0,5.0	7.0,0	7.0,7.0	0,7.0	5.0,0	5.0,5.0	0,5.0
BORNEU09	ΔQ_x	-0.25	0.67	0.35	-0.31	0.87	2.71	-2.26	-0.26	0.04
	ΔQ_y	0.84	0.04	-0.51	0.45	5.00	-0.39	1.72	0.88	0.48
BORNEU21	ΔQ_x	-1.06	5.90	5.75	-2.10	6.76	-2.51	18.5	3.24	-9.09
	ΔQ_y	7.00	2.40	-3.86	7.87	-4.64	-0.63	-1.61	-1.94	6.80
BORNEU32	ΔQ_x	-4.41	0.60	-0.53	-6.09	0.94	9.17	-20.6*	-1.18	4.78
	ΔQ_y	3.90	0.19	-0.19	6.51	1.41	-8.56	15.8*	4.02	-1.80
BORGAU09	ΔQ_x	-1.41	0.78	1.61	-0.08	0.91	2.43	-0.89	-0.72	-1.67
	ΔQ_y	1.82	1.76	-0.22	-0.13	5.76	-0.23	0.00	-0.91	0.30

* Note that the FFT program misidentified the two tunes, giving the unphysical result that the tunes switched. In any case, the tolerance is exceeded.

The case BORNEU320 is not shown. While it was fully satisfactory at small amplitudes, at large amplitudes particles were lost during tracking. Clearly a period of nine or less is acceptable for very large amplitude particles, while twenty-one or greater is not.

8. Investigation of the Possibility of Permitting Weak Lumped Correctors Only at Spool-Piece Locations.

8.1 General Comments

In the Tevatron all correction elements are located in "spool-pieces" which are situated immediately next to main arc quadrupoles. It is natural to contemplate a similar configuration for the SSC, and it would be more economical to include multipole correctors in those locations than in the cell interior. For that reason, considerable effort was expended in attempting to achieve satisfactory systematic compensation without the use of interior elements. The results are indicated in the following tables; small-amplitude and large-amplitude results are given in different tables, in the expectation that systematic small-amplitude behavior can be compensated satisfactorily, but that large amplitude behavior cannot be. As usual, "large" amplitude is taken to mean maximum excursion of 5mm, on-momentum, and 3.6mm, off-momentum.

Unless stated otherwise, the assumed dipole systematic errors are: $b_2 = 0.0$ (by virtue of full-length bore-tube correction assumed to be present,) $b_3 = 0.1, b_4 = 0.64, b_6 = -0.13$, and all other multipoles 0.0. Remote correction was in the spool-pieces of every tenth cell. Also (to see if remote and local compensation are equivalent) results are given with elements in every cell, and with b_3 or b_6 set to zero (to see if they are negligible.)

Multipole corrector strengths are indicated in the tables as ratios of their strength-length product to the field integral of the corresponding multipole in the dipole; with all values being 1, the field integrals around-the-ring of each dipole multipole and its corresponding compensators would be equal and opposite; were the compensators superimposed exactly on the errors this would obviously yield perfect compensation. Since, in fact, the spool-pieces are at points where the β-functions are large, the optimal values tend to be less than 1.

The correctors can be said to be grouped in four families, with strength parameters F_3 and F_4 for the elements beside F-type quads, and D_3 and D_4 for the elements beside D-type quads.

8.2 Small Amplitude Behavior.

SPOOL-ONLY COMPENSATION SMALL AMPLITUDE TUNES COARSE SCAN, TWO CORRECTOR FAMILIES							
FRACTIONAL STRENGTHS					$\delta = -0.001$	$\delta = 0.0$	$\delta = 0.001$
F_3	F_4	D_3	D_4				
1.000	1.000	1.000	1.000	Q_x	14.5	0.1	-31.3
				Q_y	0.5	0.5	0.0
0.700	0.700	0.700	0.700	Q_x	4.6	0.1	-7.7
				Q_y	2.0	0.4	-8.9
0.400	0.400	0.400	0.400	Q_x	-3.7	0.1	15.7
				Q_y	4.6	0.5	-18.1
0.600	0.600	0.800	0.800	Q_x	2.8	0.1	-0.1
				Q_y	3.3	0.5	-9.3
0.700	0.700	0.700	0.700	Q_x	4.6	0.1	-7.7
				Q_y	2.0	0.4	-8.9
0.800	0.800	0.600	0.600	Q_x	8.0	0.1	-15.5
				Q_y	1.6	0.5	-8.5

The data in the previous table show that the small amplitude tunes can almost be compensated adequately with correctors in the spool-pieces, even when the correctors are constrained to be in two families (as they would be, for example, if a single coil was designed to compensate more than one persistent current multipole). This coarse scan of the two parameters shows that the minimum is very broad; the settings are not critical.

The next table shows small-amplitude results after all four corrector-strength parameters have been adjusted for better compensation. For all data sets but the last, the parameters are adjusted to give optimal large-amplitude behavior (which is why the small-amplitude behavior is not much improved even after doubling the number of fitting parameters). Further comments, including an explanation of the parameter adjustment algorithm, will be deferred to the next section.

SPOOL-ONLY COMPENSATION SMALL AMPLITUDE TUNES PERFORMANCE WITH OPTIMIZED CORRECTOR SETTINGS							
Fractional Strengths					$\delta = -0.001$	$\delta = 0.0$	$\delta = 0.001$
F_3	F_4	D_3	D_4				
Remote, every 10'th cell. $b_6 = -0.13$							
0.681	0.619	1.035	0.517	Q_x	0.3	0.1	-3.6
				Q_y	6.7	0.5	-9.9
Remote, every 10'th cell. $b_6 = 0.00$							
0.713	0.582	1.066	0.539	Q_x	-0.3	0.1	-3.5
				Q_y	7.4	0.5	-9.0
Correction in every cell. $b_6 = 0.00$							
0.742	0.633	1.005	0.598	Q_x	0.8	0.1	-5.6
				Q_y	5.0	0.5	-7.6
OPTIMIZATION BY OPERATIONAL SIMULATION $b_6 = -0.13$							
0.563	0.470	1.365	1.800	Q_x	0.1	0.1	0.2
				Q_y	0.5	0.5	0.4

The last data set in the previous table shows the performance after the parameters have been adjusted using an operational simulation procedure which employs only information which would reasonably be expected to be operationally available on the accelerator.[9] Only small-amplitude behavior is compensated. It can be seen that this procedure results in the small amplitude specification being met by a large factor.

8.3 Large Amplitude Behavior.

SPOOL-ONLY COMPENSATION LARGE AMPLITUDE TUNES COARSE SCAN, CONSTRAINED RATIOS														
Fractional Strengths					$\delta = -0.001$			$\delta = 0.0$			$\delta = 0.001$		
F_3	F_4	D_3	D_4		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
1.000	1.000	1.000	1.000	Q_x	28.5	31.6	15.8	-5.7	-7.8	-0.1	-73.5	-69.4	-32.2
				Q_y	1.6	8.4	6.5	-0.3	-6.7	-6.7	-0.5	-12.7	-12.6
0.700	0.700	0.700	0.700	Q_x	8.5	19.6	15.7	-0.2	-6.1	-4.1	-19.5	-32.3	-23.7
				Q_y	12.0	11.0	0.7	-4.0	-5.2	-0.5	-22.8	-22.8	-7.1
0.400	0.400	0.400	0.400	Q_x	-11.5	7.0	15.4	4.0	-4.0	-8.0	25.6	0.1	-11.6
				Q_y	20.4	12.5	-3.4	-7.7	-3.6	4.4	-42.5	-30.8	-3.8
0.600	0.600	0.800	0.800	Q_x	3.3	13.0	12.6	0.3	-3.8	-4.1	-5.5	-19.4	-15.5
				Q_y	12.4	13.3	4.1	-4.0	-7.1	-3.1	-23.1	-26.9	-11.9
0.800	0.800	0.600	0.600	Q_x	15.7	25.1	16.6	7.0	-7.5	-4.2	-35.0	-46.6	-27.7
				Q_y	10.0	8.0	-2.1	-4.0	-3.4	0.6	-22.0	-18.5	-4.3
$b_3 = 0.00$													
0.700	0.700	0.700	0.700	Q_x	10.8	23.7	19.5	0.2	0.2	0.0	-19.3	-29.4	-19.3
				Q_y	17.1	16.3	4.5	0.4	0.6	0.5	-17.7	-15.3	-3.9

This table shows large-amplitude tunes obtained in a coarse scan with the parameters constrained to be in two families. The best case fails to meet tolerance by about a factor of four. Comparison of the second and last data sets shows that b_3 is not very important.

Next an effort was made to adjust the full four-family parameter settings in such a way as to improve the large amplitude tune behavior, with the results shown in the next table

SPOOL-ONLY COMPENSATION

LARGE AMPLITUDE TUNES

PERFORMANCE AFTER OPTIMIZED PARAMETER SETTINGS

Fractional Strengths					$\delta=-0.001$			$\delta=0.0$			$\delta=0.001$		
F_3	F_4	D_3	D_4		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
Remote, every 10'th cell. $b_6=-0.13$													
0.681	0.619	1.035	0.517	Q_x	0.1	15.8	15.8	-0.2	-3.7	-3.3	-7.7	-24.3	-20.0
				Q_y	20.3	11.7	-3.3	-3.1	-8.2	-7.0	-26.7	-23.0	-6.6
Remote, every 10'th cell. $b_6=0.00$													
0.713	0.582	1.066	0.539	Q_x	0.3	15.2	14.5	-0.7	-3.7	-2.9	-11.6	-26.4	-19.4
				Q_y	19.9	11.8	-3.2	-2.9	-8.5	-7.2	-23.1	-22.8	-6.9
Correction in every cell. $b_6=0.00$													
0.742	0.633	1.005	0.598	Q_x	7.8	18.9	14.8	-1.5	-3.9	-3.3	-12.8	-27.1	-20.7
				Q_y	15.2	10.5	-2.1	-3.1	-7.7	-6.6	-21.9	-22.3	-7.1
OPTIMIZATION BY OPERATIONAL SIMULATION $b_6=-0.13$													
0.563	0.470	1.365	1.800	Q_x	7.0	3.7	3.2	0.1	-2.0	-1.5	-2.9	-4.0	-3.8
				Q_y	0.7	35.3	31.9	-2.1	-18.8	-16.4	-3.1	-50.6	-46.5

To obtain these fits, a parameter fitting option was added to TEAPOT which calculates the "large-amplitude interpolated map"[15] as the parameters are varied and finds the settings which makes the "worst map matrix element" be as "good" as possible, where good means "close to the small amplitude element". This is not precisely the same as optimizing the standard tracking cases but should be similar. Also it is not necessarily achievable operationally; for that reason it might not be prudent to rely on. In any case, the results are given in the table, for the same cases as were listed in the small-amplitude table above. It can be seen that there is not much improvement. It can also be seen that b_6 is unimportant, and that remote and local compensation do not differ in any important way.

Because the large amplitude behavior was not much improved another parameter fitting algorithm was tried. It paid attention only to the tunes, both x and y, and adjusted the parameters to "keep the tunes in as small a box as possible". (The standard specification for tune variation is to stay in a box of height 10 units (i.e. a range of ± 0.005) along the tune axes, for all momentum deviations less than $\pm$ 0.001.) For technical reasons this is still not equivalent to optimizing the standard tracking cases, but it's close. The results, shown in the next table, are no better than before.

SPOOL-ONLY COMPENSATION
LARGE AMPLITUDE TUNES
PERFORMANCE AFTER ALTERNATE OPTIMIZATION

Fractional Strengths					$\delta = -0.001$			$\delta = 0.0$			$\delta = 0.001$		
F_3	F_4	D_3	D_4		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
0.776	0.649	0.865	0.788	Q_x	8.1	17.3	12.2	-2.7	-5.0	-3.7	-15.6	-27.8	-17.0
				Q_y	12.5	11.7	0.6	-3.3	-7.1	-3.4	-19.6	-22.5	-7.6

The large amplitude behavior after operational compensation[9] (briefly introduced in the previous section) of the small-amplitude behavior is unfortunately large. It was not unexpected that large and small amplitude behavior would be essentially different[6] so these numbers are not surprising. They can perhaps be used to make the following not-very-quantitative projection.

If the systematic field errors were unknown, or were uncertain by amounts comparable with the multipoles used in this study, the small-amplitude behavior could still be compensated operationally, presumably rather well, as indicated in the previous section. But in that case one would have to expect large-amplitude shifts comparable with the values in the last row of this table; these are "outside specs" by close to a factor of ten.

8.4 Relaxing the Zero Chromaticity Condition.

Another thing that was tried was to "cheat", by letting the chromaticity be not quite zero, in an attempt to stay within the tune tolerance box. The results are shown in the next table.

SPOOL-ONLY COMPENSATION, CHROMATICITY ADJUSTED LARGE-AMPLITUDE TUNES										
F, D correction,%	tune	-0.001			0.0			0.001		
		3.6,0	3.6,3.6	0,3.6	5.0,0	5.0,5.0	0,5.0	3.6,0	3.6,3.6	0,3.6
70,70	Q_x	-11.4	0.0	-4.1	-0.2	-5.1	-4.1	0.1	-12.1	-2.6
70,70	Q_y	-3.3	-3.8	-14.7	-4.3	-5.3	-0.5	-7.3	-7.3	5.9

It is very nearly possible to remain within the box. This illustrates the sort of maneuver which might be attempted with the accelerator in place but which should surely not be counted upon in the design phase. It happens, in this case, that the little bit of linear chromaticity added has the same positive sign which would be required to stabilize the beam against the head-tail instability.

8.5 CONCLUSIONS.

Valiant but unsuccessful efforts have been made to compensate the higher multipole systematic tune behavior using compensation elements located only in the spool-pieces. It has to be concluded either that the systematic field errors must be reduced (for example by improving the magnets or by raising the injection energy) or that correction elements must be situated in cell interiors.

9. Random Error Correction Performance

9.1 General Comments

The transverse amplitude and momentum dependence due to random multipole errors in the dipoles has been examined for the various correction schemes, all of which satisfied the systematic error requirements. For any lumped compensation elements it is assumed that "binning"[16] circuits are available; results with 7 bins and with infinitely fine bins (that is, the corrector strengths can all be adjusted independently of each other) were obtained. No closed-orbit errors will be introduced until the next section.

It will be shown that each of the main correction schemes is capable of compensating for the random errors specified for the SSC. Smear values of about 5% are obtained after compensation of only random b_2 errors. This represents an improvement of more than a factor of two compared to the uncompensated situation. Compensation of higher multipoles could give roughly another factor of two in smear reduction if implemented.

This section can in some ways be regarded to be a continuation of an earlier report;[17] further explanations and references are given there, as well as the performance of other schemes not described here. A constraint imposed early on was that no more than two physical correctors be present per half cell.

Random errors dominate the smear in the SSC. The dipole errors included in the study are the random sextupole, octupole and decapole errors, including their skew counterparts; values are shown in Table 1.

Table 1. RMS Variations of Multipole Errors in SSC Dipoles ($10^{-4}B_0$ at $1cm$).[18]

a_2	0.6	b_2	2.0
a_3	0.7	b_3	0.3
a_4	0.2	b_4	0.7

The four schemes studied have been described in earlier sections. Their code names are repeated here (after being slightly abbreviated; BOR has been reduced to B; it still denotes schemes with bore-tube coils): BCDR has partial-length bore-tube correctors b_2, b_3, and b_4; BFUL5 has full-length bore-tube b_2 correctors and lumped correctors (of Simpson-Neuffer type) every fifth cell; SNEU and GAUI depend entirely on lumped correction (of Simpson-Neuffer and Gaussian integration type respectively.)

Because of statistical fluctuations in the assumed errors, it is difficult to obtain accurate aperture determinations. To improve inter-scheme comparisons the same field errors were assigned where possible; that is, for BCDR, BFUL5, and SNEU. Since GAUI requires a different magnet configuration, it is less easily compared. For GAUI the total number of magnets differs from the other schemes by a factor of 5/6 or 10/6. For this investigation we have assumed 5/6 (and have retained the same cell-length as for the other lattices), even though that is almost certainly not a practical combination, since it would use longer magnets. This was done to make comparisons as direct as possible. The same r.m.s multipole coefficients (Table 1) were used in all cases; this effectively increases the r.m.s. multipole errors for GAUI by an artificial factor $\sqrt{6/5}$ compared to the others. That will be seen below to cause GAUI to have a smaller aperture than the other schemes; this difference should be allowed for mentally in making comparisons.

The corrector strengths of the two lumped schemes were evaluated by applying the general scheme developed by Forest[19,20] to the individual half cell of six bending magnets and two correctors. We obtain for SNEU and GAUI the following corrector strengths.

SNEU Scheme (in "three lumped corrector representation"):

$$
\begin{aligned}
C_f L_f &= \frac{L_b}{108}[-83\alpha_1 - 41\alpha_2 - 11\alpha_3 + 7\alpha_4 + 13\alpha_5 + 7\alpha_6] \\
C_c L_c &= -\frac{L_b}{27}[8\alpha_1 + 20\alpha_2 + 26\alpha_3 + 26\alpha_4 + 20\alpha_5 + 8\alpha_6] \\
C_d L_d &= \frac{L_b}{108}[7\alpha_1 + 13\alpha_2 + 7\alpha_3 - 11\alpha_4 - 41\alpha_5 - 83\alpha_6]
\end{aligned}
\tag{9.1}
$$

GAUI Scheme:

$$
\begin{aligned}
C_1L_1 =& \frac{L_b}{6}[-7\alpha_1 - 5\alpha_2 - 3\alpha_3 - \alpha_4 + \alpha_5] \\
C_2L_2 =& \frac{L_b}{6}[\alpha_1 - \alpha_2 - 3\alpha_3 - 5\alpha_4 - 7\alpha_5]
\end{aligned}
\tag{9.2}
$$

where α_i is the multipole error of the i^{th} dipole magnet. C_xL_x is the integral strength of corrector x. Note that the weights do not depend on the multipole order. The correction strengths for the bore tube based correctors (BCDR, BFULx, where x is a number indicating the period in units of cells of the remote correctors) are determined by the integral strength of the multipole moments within each magnet.

To obtain the amplitude dependence of the smear, on-momentum particles were tracked around the "arcs only" SSC lattice. "Worst-case" particles, having equal "single particle emittances," were tracked. These particles lie along a 45 degree line at points in the lattice where β_x and β_y are equal. In some earlier CDG reports the axes of graphs have been labelled with $\sqrt{(x_{max}^2 + y_{max}^2)}$ in millimeters, where x_{max} and y_{max} are amplitudes at a point in the lattice where the corresponding β-function is maximum (of course there is no point where these functions are simultaneously maximum). References to the graphs should emphasize the $x = y = 5$ mm point, since that defines the on-momentum "needed-aperture". (The detailed estimate yielded a needed aperture of $x = y = 4.7$mm; the "needed-aperture" is rounded upwards from this number.)

The tune variations $\Delta Q_x(x, y, \delta)$ and $\Delta Q_y(x, y, \delta)$ after correction for the random errors are in general much smaller than the allowed maximum tune shift variation of ± 0.005. The results will therefore not be presented explicitly.

Prior to actual tracking, systematic errors are compensated as described in earlier sections and random errors are corrected by the above described formulas. Tracking results are Fourier-analysed to obtain the tune and smear.

For the main study, where just b_2 errors were compensated, a total of 5 seeds are run for each of the correction schemes in addition to the uncorrected case. In the study of the contribution to the smear by the individual skew multipole errors, only two seeds were used. To achieve adequate accuracies for the tracking results, the SSC dipoles are represented by two kicks per dipole. Studies on the lattice with systematic errors have shown that this provides acceptable accuracy.[11]

Random errors were assumed to be Gaussian distributed. For some early runs 2σ cuts were applied to all multipoles; magnets falling outside the cuts were rejected. But respecting such cuts would require extreme discipline which, as a practical matter, might not be adhered to in the heat of magnet installation in the tunnel. For most of the subsequent runs 6σ cuts were used.

The binning correction goes as follows. For any particular multipole, the numerical values of that multipole are ordered and "percentile" boundaries are defined which divide the magnets up equally (in 7 equal groups for most of the subsequent data). Every magnet in the same bin for the particular multipole gets the same "central to the bin" compensation for that multipole; initially the bin half-way point was used. An effect of increasing the cuts was to make the binning compensation less effective, since the outermost bins had to go out to 6σ. It was found that the worst degradation came from the outermost bins. Using the median of the values actually falling within each bin, rather than the bin centers, yielded much better results and that was done for all the 6σ data.

9.2 Smear Contributions From Higher Multipoles, 2σ Cuts.

The next table shows the smear observed, on-momentum, with and without b_2 compensation, as higher multipoles are one-by-one increased from zero to their expected distributions (Gaussian, cut at 2σ). In the table the numerator of each entry is the smear with b_2 7-bin correction, and the denominator is the smear without this correction and everything else the same. The value of the fraction gives the factor by which the correction reduces the smear. The smear is given in percentage, thus the CDR acceptable tolerance is 10.

PERCENT SMEAR AT 5mm, 5mm Only 2 seeds averaged, 2σ cuts 7-bin, b_2 corrected smear/uncorrected smear						
	syst. only	$+b_2$	$+a_2$	$+b_3$	$+a_3$	$+b_4+a_4$
SNEU	0.75/0.75	2.4/7.4	3.2/8.4	3.3/8.6	4.4/9.3	4.2/9.1
GAUI	0.19/0.19	3.1/11.4	4.8/12.5	5.2/13.1	5.7/13.7	5.5/13.7
BFUL5	0.36/0.36	1.2/6.5	2.4/8.0	2.5/8.0	3.5/8.3	3.5/8.5
BCDR	0.33/0.33	1.4/6.6	2.4/7.7	2.5/7.6	3.8/8.1	3.9/8.4

It is evident from the table that all schemes pass the CDR tolerance. With only b_2 errors the schemes with bore-tube b_2 correctors, BFUL5 and BCDR, do much better than the others, but when higher multipoles are included the advantage is largely eroded. Fig. 9.1 illustrates qualitatively the amplitude dependence of the smear for one seed of the BCDR compensation scheme with increasing order of random multipole errors present. After the erect sextupole, the most significant degradations of the smear occur with the addition of the random skew sextupole and octupole errors. The remainder of higher order multipoles including the decapole errors (not plotted) have a minimal effect on the result.

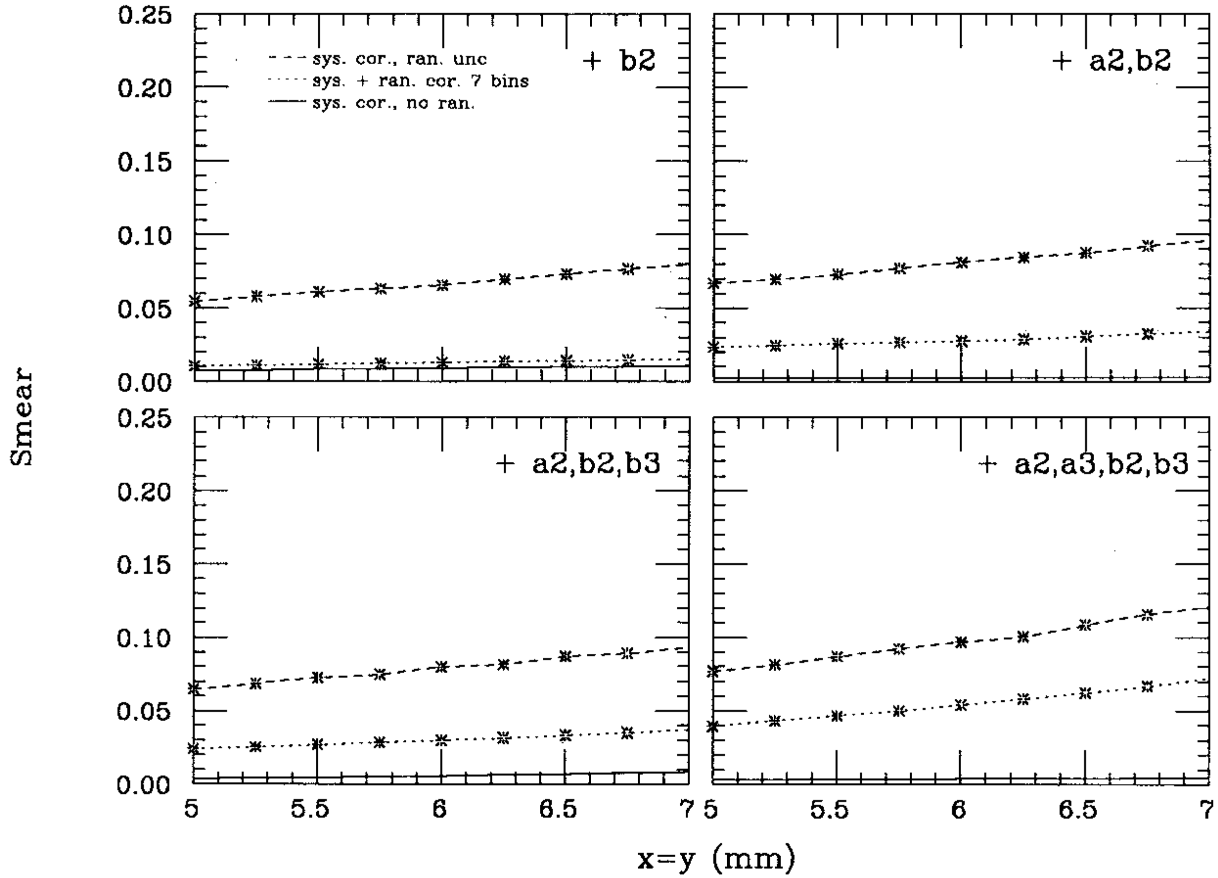


Fig. 9.1: Smear vs Amplitude for random error compensation and increasing order of random multipole errors. The smear before and after correcting for the random b2 is plotted for one seed of the BCDR compensation scheme. The cutoff for the random errors present (upper right hand corner of each plot) was set at 6σ. The description of the individual curves are given in the first figure.

9.3 Amplitude Dependence With Better Statistics and 6σ Cuts

In the next table, smear values, on-momentum, with and without b_2 correction, are given as in the previous table. This time all multipoles are assumed to be present and the smear is determined for amplitudes of (5mm,5mm), (6mm,6mm), and (7mm,7mm). The values are averages of 5 seeds. Estimates of the r.m.s. errors were worked out from the 5 numbers; they are not entered in the table, but they are plotted in the following graph (Fig. 9.2), which includes the data from the following two tables.

With the larger amplitude and looser cuts some particles in this investigation did not survive the 512 turns for which they were to be tracked. (The number of turns was chosen to provide adequate data to compute the smear. In one case studied, at 512 turns the smear had attained 99.7% $\pm$ 0.1 of the smear value obtained from 2048 turns.) In the table this is indicated by a number in parenthesis which gives the number of surviving particles. The average listed is the average of the surviving particles; as such it is pretty much useless. In all cases but one, all particles at all amplitudes survived after correction. That one case was for GAUI which, as mentioned previously, having fewer magnets, has artificially increased smear.

As previously, the factor by which the smear is reduced by the correction is given by a ratio. Recall that different seeds are used for GAUI. That, and the smaller number of magnets, accounts for the worse uncompensated smear of GAUI, and that accounts for its worse compensated smear. The factor by which the compensators reduce the smear is best for GAUI, but that is probably because the starting value is worst.

PERCENT SMEAR AT VARIOUS AMPLITUDES 5 seed averages, 6σ cuts 7-bin, b_2 corrected smear/uncorrected smear			
	5mm,5mm	6mm,6mm	7mm,7mm
SNEU	5.4/10.1	7.2/(0)	9.9/(0)
GAUI	6.6/15.1	10.1/(0)	12.0(4)/(0)
BFUL5	4.6/9.4	6.4/14.1(5)	8.6/15.1(5)
BCDR	4.8/9.5	6.5/(0)	8.5/(0)

The next table is the same as the previous one except that perfect binning is assumed, rather than 7 bins. That is equivalent to having an infinite number of bins and is also equivalent to having the ability to power every b_2 coil individually.

PERCENT SMEAR AT VARIOUS AMPLITUDES INFINITE BINS 5 seed averages, 6σ cuts ∞-bin, b_2 corrected smear/uncorrected smear			
	5mm,5mm	6mm,6mm	7mm,7mm
SNEU	4.7/10.1	6.5/(0)	9.0/(0)
GAUI	5.9/15.1	8.5/(0)	13.9/(0)

There is not a great improvement, compared to the 7-bin data; earlier data of this sort (and a dash of common sense) was the basis for choosing 7 bins.

The following figure presents pretty much the same information in graphical form. It can be seen that the error bars resulting from the small number of seeds make the absolute values rather uncertain. Fortunately, for comparing different schemes relative values suffice.

9.4 Conclusions to This Point

To this point in the study, the candidate lattices BCDR, BFUL5, SNEU, and GAUI have satisfied the requirements first of systematic compensation and now of random compensation. In some ways performance of one or the other has been found to be measurably superior (e.g. the remote schemes tend to be slightly worse than the local schemes), but the differences are small, probably not great enough to stack up against qualitatively different considerations like cost and practicality. In the next section the more delicate issue of sensitivity to closed orbit errors will give a greater discrimination among the schemes.

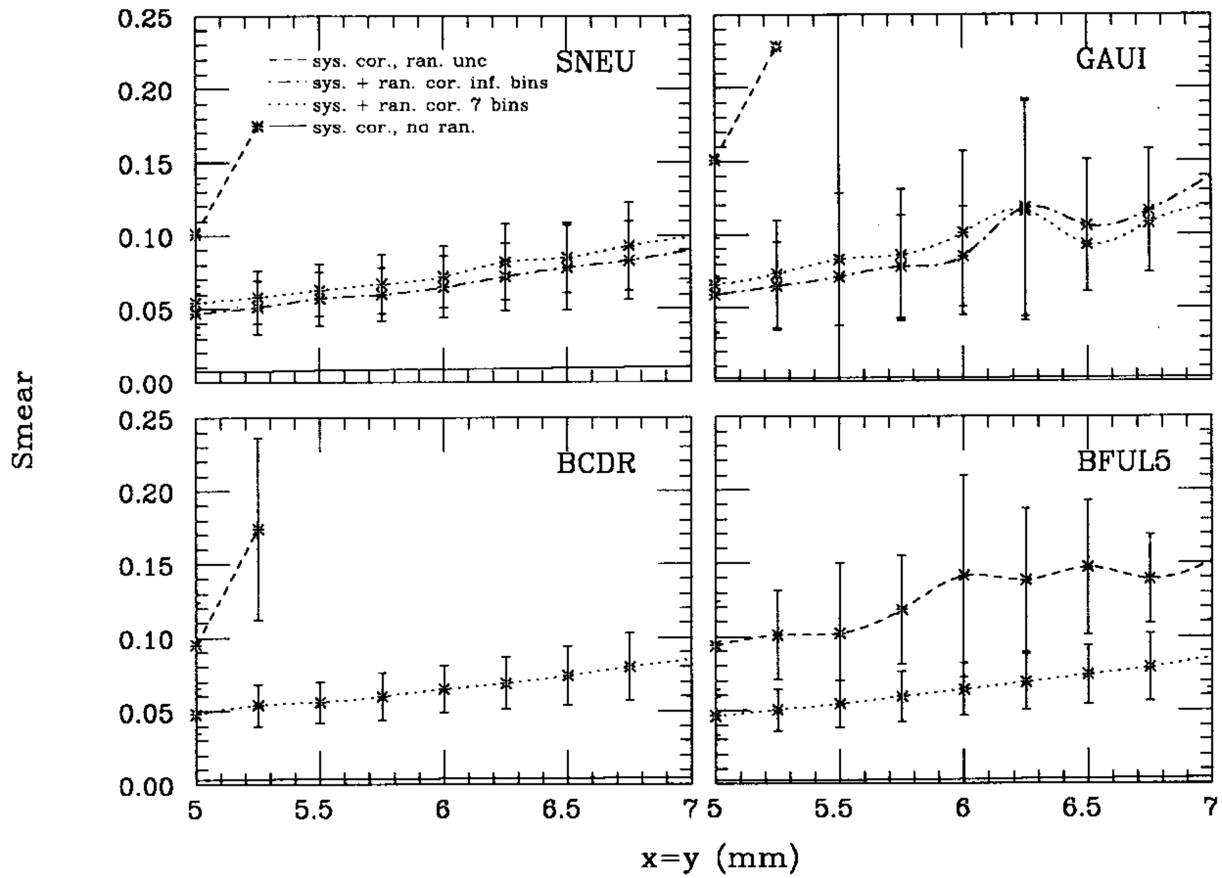


Fig. 9.2: Smear vs Amplitude for random error compensation. The 5 seed averaged smear before and after correcting the random b2 is plotted as function of the amplitude for the SNEU, GAUI, BCDR and BFUL5 compensation schemes. Points without corresponding error bars have insufficient data to calculate accurate standard deviations. The cutoff for the random errors (a2–a4, b2–b4) was set at 6σ. Particles not surviving 512 turns were not plotted. The description of the individual curves are given in the first figure.

10. Effects of Closed-Orbit Errors.

We now describe early results of a study of the sensitivity of four of the schemes studied so far to the inclusion of closed-orbit errors. The results are very preliminary – only one seed has been studied so far, only the on-momentum behavior has been investigated, and random multipoles have not yet been included. More work is necessary before these results can be considered fully reliable.

The same four schemes studied in the previous section on random error correction were used to study sensitivity to orbit errors caused by quadrupole magnet misalignment, dipole magnet rotation and misalignment, and dipole magnet field errors. The correctors were not misaligned. These studies were conducted with only systematic multipole errors, no random multipole errors, in the hopes that interpretation of the results would be made simpler.

BORCDR, BORFUL5, SNEUFF and GAUINT were all prepared in the following way: systematic multipole errors were added and the correctors were set to compensate them; the alignment and field errors mentioned above, with strengths adjusted to produce the desired residual closed orbit errors, were added and the orbit was corrected, leaving a $\pm$1mm r.m.s. orbit; the tunes and linear chromaticities were set; the resulting machines were tracked for 512 turns, with the smears and tunes shifts being measured for various amplitude particles, all on-momentum. To date, only one random seed has been studied. It is important to remember while looking at the tracking results, that the needed aperture of 5mm decreases by approximately 1.25mm when orbit errors are present in the machine being tracked. That is, part of the needed aperture is for orbit errors, so if they are included in the simulation, they can be subtracted from the needed aperture.

Although the machines studied had only systematic multipole errors, we expect the smear to be non-zero since the orbit errors will cause randomness in the feed-down of the systematic multipoles, which has the same effect as random multipole errors. The smears and tune shifts for BORCDR and BORFUL5 are given in the following Table. The behavior of BORFUL5 is worse than BORCDR, but it is still acceptable; further comments appear later in this section. In this case, and for this seed, BORFUL5 is acceptable since the smear is less than 10% at (5,5), but the smear for a real machine with random errors, misaligned dipoles, etc. would certainly be larger.

SMEARS AND TUNE SHIFTS WITH ORBIT ERRORS SYSTEMATIC ERRORS ONLY						
	X (mm)	Y (mm)	Q_x	Q_y	Smear$_x$	Smear$_y$
BORCDR	0.0	0.0	0.2852	0.2653	0.0	0.0
	3.0	3.0	0.2851	0.2653	1.0	1.5
	4.0	4.0	0.2851	0.2654	1.5	2.6
	5.0	5.0	0.2851	0.2654	2.2	3.9
	6.0	6.0	0.2850	0.2655	2.9	5.4
BORFUL5	0.0	0.0	0.2851	0.2655	0.0	0.0
	3.0	3.0	0.2848	0.2656	2.3	2.6
	4.0	4.0	0.2842	0.2657	4.0	4.3
	5.0	5.0	0.2836	0.2661	6.5	6.5
	6.0	6.0	0.2835	0.2675	9.9	9.4

When a similar study of SNEUFF and GAUINT was attempted, it was not possible to fit the tunes to the nominal values due to linear coupling, presumably from the feed-down of systematic b_2. The distance of closest approach of the tunes was: SNEUFF, 0.054; GAUINT, 0.063; BORCDR, 0.013; BORFUL5, 0.006. Rather than pursue ways of overcoming this problem operationally it was decided to "turn off" the problem temporarily. When b_2 was turned off in SNEUFF and GAUINT, the distance of closest approach of the tunes was reduced to 0.007 for SNEUFF and 0.003 for GAUINT. This is equivalent to placing a full-length b_2 bore tube corrector in each dipole. Note that SNEUFF in this case is equivalent to

BORFUL1, or a BORFUL case with lumped correctors every half-cell. Further study is necessary to determine whether these machines could be sufficiently decoupled using skew quadrupole correctors. One or the other of, decoupling using skew quads, or, installing full-length bore tube correctors, will have to be done in the accelerator. The resulting machines were then tuned and tracked, yielding the acceptable results shown in the following table. Note that these smears are comparably small with the smears from BORCDR above. Caution is advised in making any detailed comparisons, however, since only one random seed was used, and the closed orbit is different for each machine simulated.

SMEARS AND TUNE SHIFTS WITH ORBIT ERRORS $b_2 = 0$						
	X (mm)	Y (mm)	Q_x	Q_y	Smear_x	Smear_y
SNEUFF	0.0	0.0	0.2850	0.2650	0.0	0.0
	3.0	3.0	0.2851	0.2655	1.2	0.9
	4.0	4.0	0.2852	0.2656	1.8	1.4
	5.0	5.0	0.2852	0.2656	2.8	2.0
	6.0	6.0	0.2652	0.2657	4.0	2.9
GAUINT	0.0	0.0	0.2851	0.2653	0.0	0.0
	3.0	3.0	0.2850	0.2652	0.9	1.1
	4.0	4.0	0.2849	0.2650	1.4	1.7
	5.0	5.0	0.2847	0.2646	2.0	2.5
	6.0	6.0	0.2841	0.2641	2.8	3.5

10.1 Further study of BORFUL5.

In an attempt to gain further understanding of the larger smears for BORFUL5 shown in the first table in this section, several runs were done in which the only random errors introduced were misalignments of the lumped correctors themselves. In these cases, tracked particles went through the center of all elements except the lumped correctors (which correct only b_3 and b_4), which received ± 1 mm r.m.s. misalignments. The results of tracking this lattice are shown in the following table.

SMEARS AND TUNE SHIFTS ONLY REMOTE CORRECTORS MISALIGNED						
	X (mm)	Y (mm)	Q_x	Q_y	Smear_x	Smear_y
BORFUL5	0.0	0.0	0.2850	0.2655	0.0	0.0
	3.0	3.0	0.2853	0.2654	2.1	1.6
	4.0	4.0	0.2865	0.2653	3.0	2.7
	5.0	5.0	0.2871	0.2651	5.1	4.1
	6.0	6.0	0.2883	0.2648	6.7	5.6

This shows that misaligning the remote correctors alone causes substantial smear, and accounts for most of the smear in the first table in this section. The difference between the smear from misaligned correctors and that from full-blown orbit errors for BORFUL5 is comparable to the smear for BORCDR. Detailed comparisons are not possible since different random seeds have been used for misaligning the correctors and creating orbit errors. A study using several seeds is necessary before any real conclusions can be drawn.

Tracking with horizontal amplitude only was also performed for comparison with the analytic calculation of feeddown from the misalignment of the remote correctors in Appendix G. The results are in very rough agreement with that calculation, which predicts a smear of approximately 5% at 5 mm. Further analytic work is needed to understand and confirm the results in this chapter. If possible, other simulation programs, such as the analytic mapping technique based on differential algebra currently being developed at CDG, should be brought to bear on this problem to check and expand on the results above.

11. Strength Estimates and Preliminary Engineering Considerations.

11.1 Preliminary Engineering Considerations.

The correction schemes that have been studied so far require either (or both) distributed bore-tube correction magnets or lumped correction magnets. The R&D on the actual proposed magnetic correction devices has lagged, so that at this time the performance of the devices is not known. Bore tube correction magnets have been the subject of a significant engineering effort at Brookhaven National Laboratory; the slow rate of progress, up to now, in that effort has been one of the driving inspirations to investigate fully lumped correction schemes. The development of bore-tube correctors is much advanced over that of the lumped correctors, however, and it is fair to say that one reason to prefer the lumped correction scheme is that, since little work has been done, there have been no failures. Nevertheless, both distributed and lumped correctors have been made to work at other accelerators, and in association with other projects.[21–23]

In this section only engineering considerations for lumped correctors will be considered. The question to be addressed here is whether strong correctors can be built in a compact, reliable, and practical manner, or will construction considerations place restrictions on the correction scheme adopted. By lumped correctors here is meant any corrector not incorporated in the main dipole construction. Lumped correctors include steering dipoles, trim quadrupoles, chromaticity sextupoles, and possibly the sextupole, octupole, and decapole correctors for dipole imperfections.

A starting point for engineering considerations is the correctors design for the Tevatron. The Tevatron correctors are random wound, epoxy-potted magnets with various multipoles radially nested. These correctors work well if currents are kept at or below approximately 25% of conductor short sample. This low percent is due to the many field configurations possible, and thus the many possible force vectors on the conductor, when the nested correctors are powered at various currents and polarities. The strengths of the SSC correctors are considerably greater than those of the Tevatron. For example, the Tevatron steering dipole corrector strength is 0.46 T·m whereas the SSC steering dipole corrector strength is approximately 2.5 T·m.[24] Again, the Tevatron sextupole corrector strength is 0.021 T·m at a radius of 1 cm compared to various SSC estimated sextupole strengths of 0.1 to 0.2 T·m at a radius of 1 cm. The space available along the beam tube for the SSC correctors will vary with the correction scheme chosen but will be of the order of 1 m. The Tevatron correctors length is 0.76 m. To maintain a reasonable diameter for a correction package, then, the fraction of short sample, or the average current density, at which the correctors can comfortably operate will have to be increased. The Tevatron correctors operate at an average current density of 120 A/mm^2. A goal for SSC correctors is 300 A/mm^2. A prototype program has just begun in order to explore lumped corrector construction.

Figures 11.1 through 11.5 present plots of magnetic field strength versus coil inner radius for various coil thicknesses and an average current density of 300 A/mm^2. The plots assume a cosθ coil configuration and infinite permeability iron beginning at a radius of 5cm. From these plots one can determine various combinations of inner coil radius, coil thickness and coil length to provide a given corrector strength. As a simple example, using Fig. 11.1, the 2.5 T·m steering dipole with an inner coil radius of 2 cm and a length of 1 m would have a coil thickness of approximately 1.05 cm. If the length were increased to 2 m the coil thickness would be approximately 0.60 cm.

Using the plots, the steering dipole strength given above, the estimated trim quadrupole strength of 0.38 T·m[25], and the sextupole, octupole, and decapole corrector strengths given in Ref. 26 one can configure radially nested correctors. As an example a radially nested 2 m long package of the steering dipole, chromaticity sextupole (0.134 T·m), and trim quadrupole could be constructed with the following radial dimensions: steering dipole coil from 2.0 to 2.6 cm, chromaticity sextupole coil from 3.1 to 3.5 cm, and trim quadrupole coil from 4.0 to

4.3 cm. This configuration allows 0.5 cm between coils.

An overriding concern in corrector construction is the current at which correctors operate. High current magnets require high current leads which consume large amounts of refrigeration. Thus for the approximately 1600 steering dipole correctors, which must be powered independently, a low current design of about 100 A is essential. Correctors which operate in large families and thus will have few current leads can operate at higher currents.

The SSC steering dipole correctors will of necessity, then, be constructed of many turns of conductor. Epoxy potting is the only practical way to build such magnets. The goal of an average current density of 300 A/mm^2 for the dipole correctors can be approached in several ways. The improvement in j_c for Nb-Ti superconductors from the Tevatron value of 1800 A/mm^2 at 4.2 K and 5 T to the current value of 2750 A/mm^2 at 4.2 K and 5 T will provide an increase in average current density of 50% if the magnet is designed to accept the increased forces. Modest gains of 10% to 15% in conductor winding density are possible over the Tevatron random winding method.

Using kapton as the conductor insulator holds promise for allowing a significant increase in the fraction of short sample achievable. Brookhaven National Laboratory has constructed unnested epoxy-potted magnets, using bare kapton as the conductor insulation, that achieve 100% of short sample after one or two training quenches. This is to be compared to unnested Tevatron correctors, using polyimide/polyamide-polyester as conductor insulation, achieving only 80% of short sample after many ($\geq$ 10) training quenches. No Brookhaven nested magnets were fabricated. Prototype testing is needed for nested correction magnets using kapton as conductor insulation. Other insulations and epoxies also need to be tested.

Potted construction with many turns of conductor is also an option for multipole correctors other than the dipole, although it is not mandated as in the case of the dipole corrector. All the above considerations apply to any epoxy-potted corrector. Thus achieving average current densities of 300 A/mm^2 and SSC strength correctors may be possible for epoxy-potted nested correctors; however extensive prototype testing will only give the answer.

Another approach to SSC corrector design is to un-nest the correctors. As noted above the Tevatron correctors achieved 80% of short sample in unnested configuration. This is a factor of three gain over the nested configuration. Using kapton as insulation will provide more of a gain. This gain will have to offset the shortening of the correction magnets as they now complete for space along the beam tube. An attractive configuration for the SSC primary corrector package, which consists of steering dipole, trim quadrupole, and chromaticity sextupole, is to share the available length half for the dipole and half for the quadrupole and sextupole. The dipole requires potted construction but the quadrupole and sextupole do not as they are powered in families. Calculation shows that a one meter long 2.5 T dipole, using 0.020-inch conductor with a copper to superconductor ratio of 1.8 to 1, will require only 50% of short sample for a 6.5 mm thick coil. The field of the dipole is much enhanced by the closeness of iron. The quadrupole and sextupole can be constructed using a construction technique employing minicables. This technique has been used successfully at Fermilab for the construction of 1000 A low beta quadrupoles. This technique uses cable and collar techniques that can achieve high fields. Using it on nested magnets will require prototyping. However it is a feasible option. If the minicable technique does not work epoxy potting and wound conductor can be tried with possible gains in fraction of short sample achievable in the absence of the strong dipole field. Sequential placement along the beam line is an option that will be investigated in the prototype program.

In summary engineering considerations for lumped correctors should not at this time place any restrictions on the correction scheme adopted. It is felt that the strengths required for the SSC can be achieved. The strengths of the steering dipole and sextupole correctors will provide the greatest challenge. As mentioned previously a lumped correctors prototype program has begun to investigate corrector fabrication.

Figure 11.1

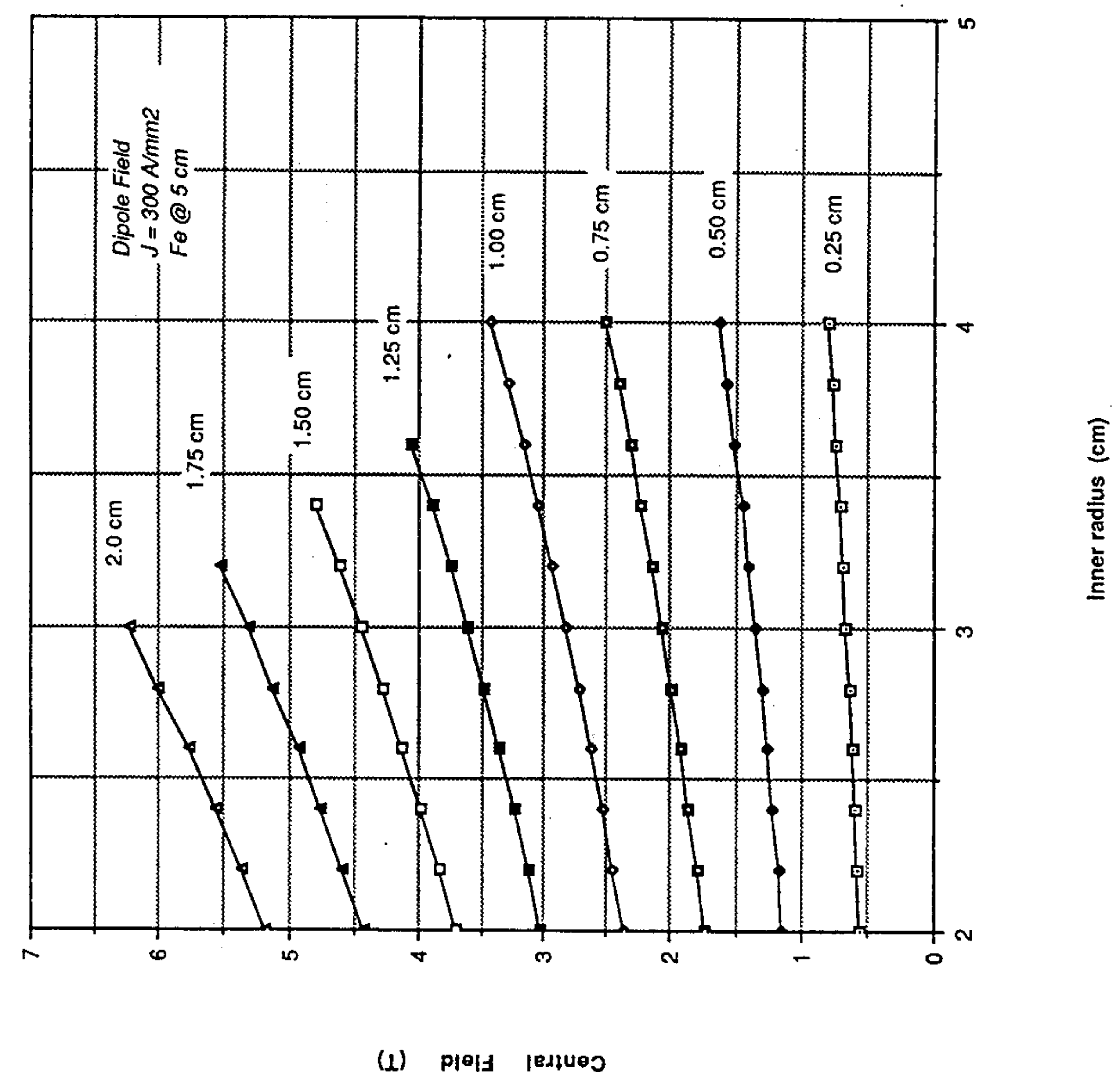


Figure 11.2

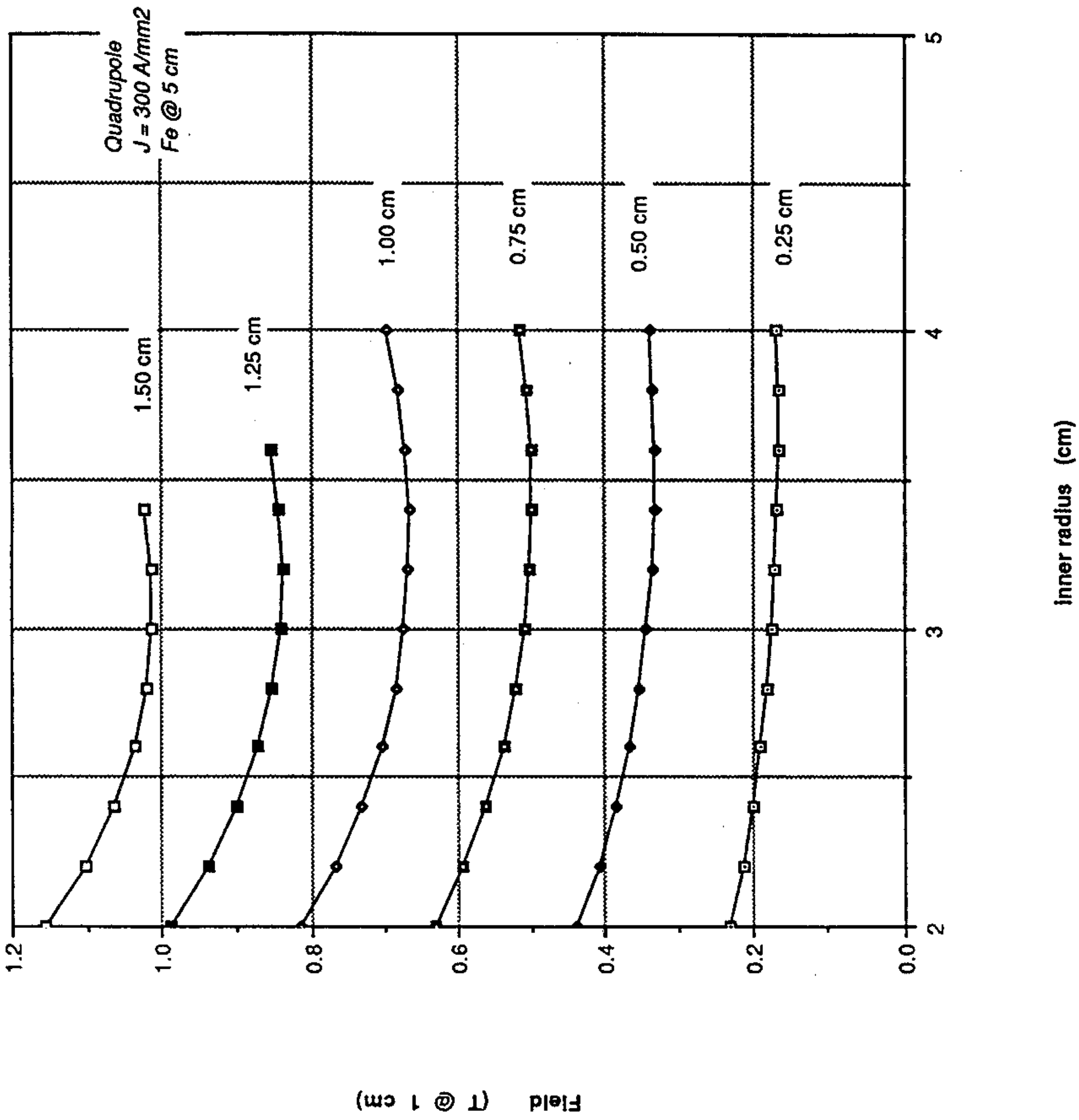
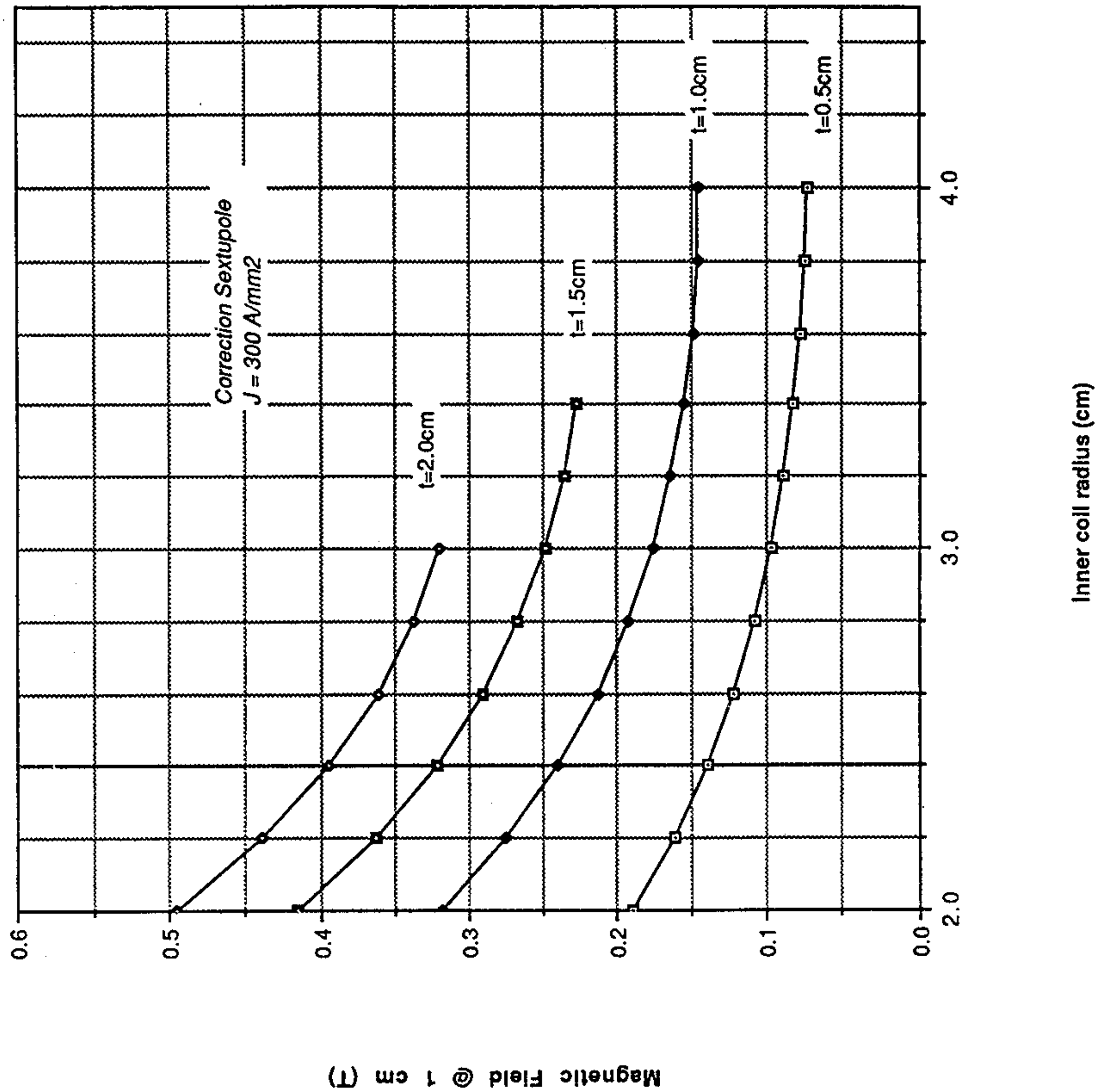
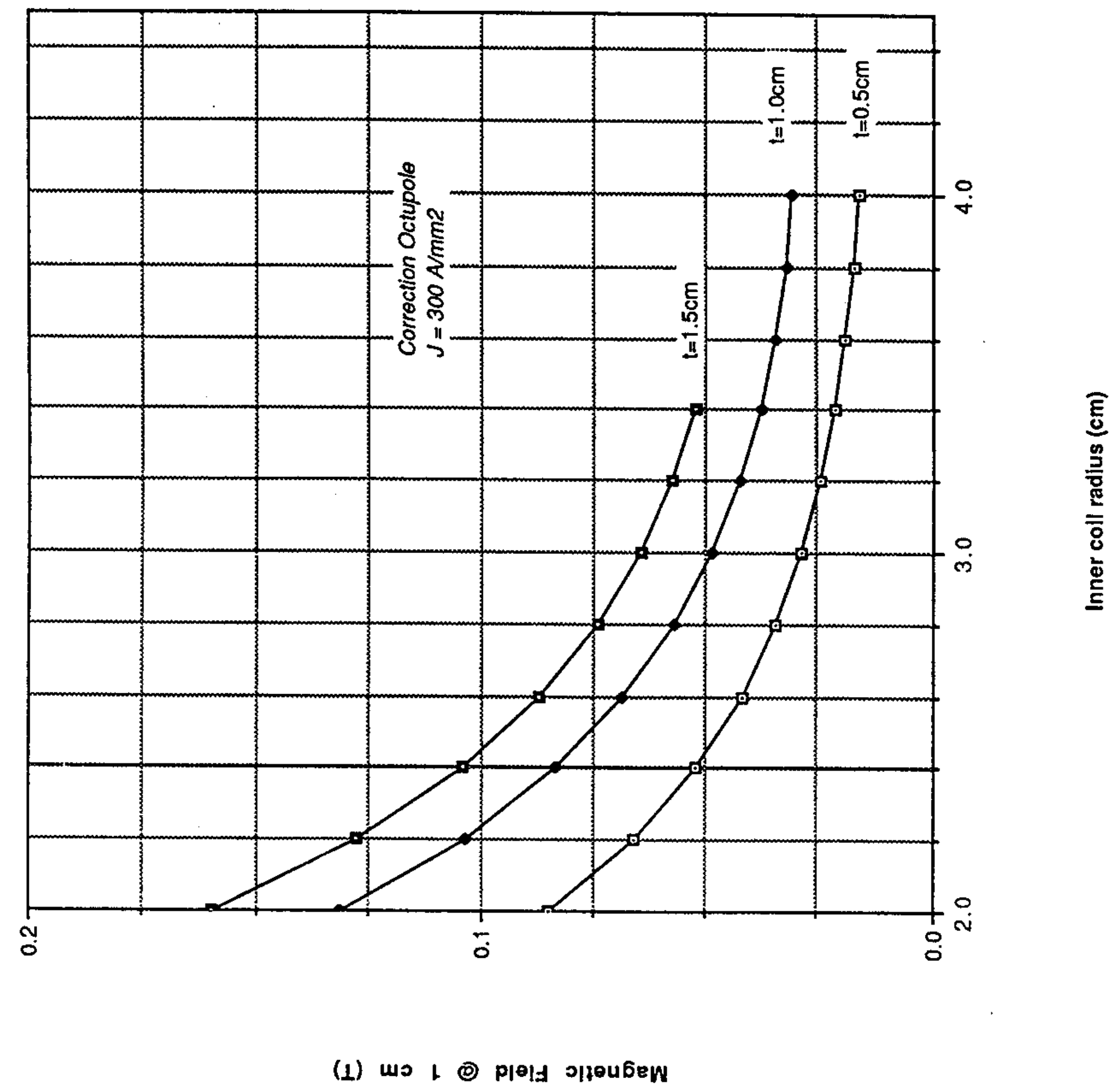

Quadrupole
J = 300 A/mm2
Fe @ 5 cm
1.50 cm
1.25 cm
1.00 cm
0.75 cm
0.50 cm
0.25 cm
1.2
1.0
0.8
0.6
0.4
0.2
0.0
2
3
4
5
Inner radius (cm)
Field (T @ 1 cm)

Figure 11.3



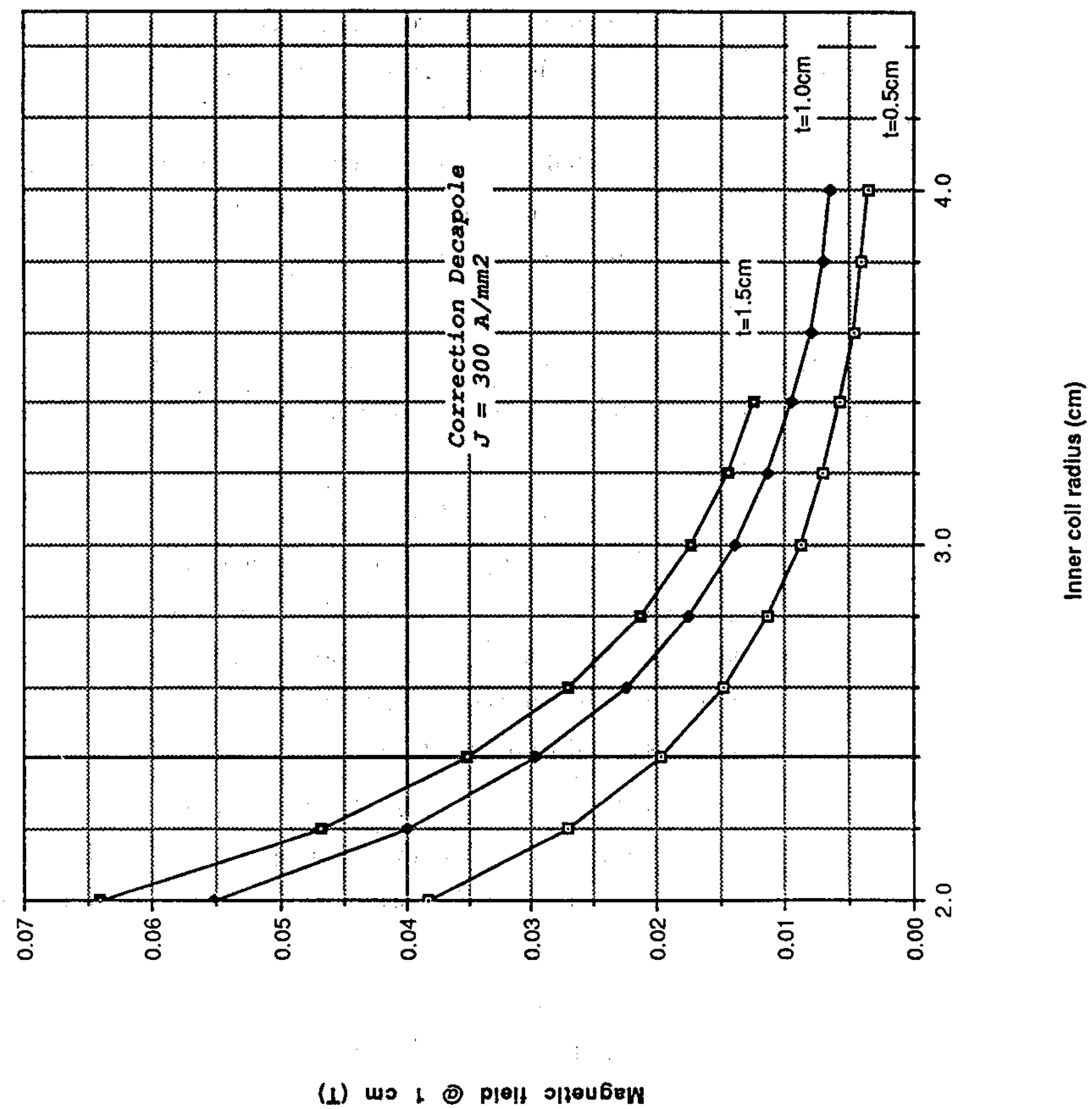
Correction Sextupole
J = 300 A/mm2
t=2.0cm
t=1.5cm
t=1.0cm
t=0.5cm
Magnetic Field @ 1 cm (T)
Inner coil radius (cm)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
2.0
3.0
4.0

Figure 11.4



Correction Octupole
J = 300 A/mm2
t=1.5cm
t=1.0cm
t=0.5cm
0.2
0.1
0.0
2.0
3.0
4.0
Inner coil radius (cm)
Magnetic Field @ 1 cm (T)

Figure 11.5


Correction Decapole
J = 300 A/mm2
t=1.5cm
t=1.0cm
t=0.5cm
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
2.0
3.0
4.0
Inner coil radius (cm)
Magnetic field @ 1 cm (T)

11.2 How Much Stronger are Dipole Correctors, Quad Trims, and Chromatic Sextupoles, When Located at "Gaussian" Locations?

In this section we discuss the impact of moving correctors from F and D locations to G locations. These letters refer to lattice locations which are defined in Appendix A. $F(D)$ labels spool-pieces beside $F(D)$ quads, and G labels points 1/5 and 4/5 along half-cells, which are close to the Gaussian quadrature points and thus give the best compensation with two correctors per half-cell. Because the number of corrector locations is being doubled, the strength at each corrector location would be halved, except that moving correctors from points of maximum β tends to reduce their "effective" strength. For the calculations that follow the required strengths will be quoted as ratios, somewhat greater than one, of the strength required at G to that required at F or D. As just mentioned, the actual strength at a G location will be reduced by a factor of two from that, because there are twice as many elements.

These strengths are needed as input to a determination of whether the improved performance of placing correctors in optimal locations justifies the cost of putting them there.

Because x-steering correctors in the CDR design are located only at points of maximum β_x, and their effective strengths are proportional to $\sqrt{\beta_x}$, when moved in-board, their required strengths are are increased by the factor $\sqrt{\beta_{xF}/\beta_{xG}} =$ 1.16. The same calculation is valid for y. Operational simulation of orbit flattening using elements at G locations (assuming that the beam position monitors remain at the quads) was found to be satisfactory.

Trim quadrupoles: for tune variation along the tune diagonal, $\delta Q_x = \delta Q_y$, which is the worst case, the F-goes-to-G required strength ratio penalty is 1.67. For tune variations orthogonal to that, the correctors no longer fight each other, and the ratio is 1.19.

For chromaticity compensation the D sextupoles are stronger than the F sextupoles (by about a factor of two) because they sit at points of low dispersion, beside vertically focusing quads. The strength penalty of moving them inboard to the G location is 1.44. For the F sextupoles the penalty factor of moving to G is 1.93, which still leaves them weaker than the other sextupoles.

12. Conclusions and Recommendations For Further Studies.

As described in the abstract, the main function of the study described in this report has been more to analyse options than to choose among them. At the same time, the shear complexity of the problem has forced us to make many non-controversial decisions in steering our way through the multi-dimensional parameter space. These have been described and will not be repeated. Here we will comment on somewhat more general questions. Some of these comments will be largely repetition, for emphasis, of points made previously in the report; others will be "indications" ("conclusions" would be too strong a word, "impressions" too weak) of expected SSC performance as it has been simulated in the investigations of this report. Some of the issues are controversial (mainly because they are difficult). For that very reason, and because they are of obvious importance, it is appropriate to review what has been learned. Since any sensible plan for future study should concentrate on further study of precisely those questions which are both important and controversial, this discussion leads naturally to recommendations for further study.

The discussion can be organized around the following questions, all of which will be more tightly specified as they are answered in the following paragraphs:

(*i*) Should field compensation be performed with distributed (bore-tube) coils, or with lumped coils?

(*ii*) Can the effects of systematic errors on global machine performance be adequately compensated?

(*iii*) Can random errors be compensated satisfactorily?

(*iv*) How well have CDR projections held up?

(*v*) What is the most critical issue the study has identified?

(*vi*) Based on simulations so far, will the operational performance of the SSC be satisfactory?

(vii) What is the "degree of conservatism" in the present design?

($viii$) What design changes are recommended?

(ix) What issues have been left out?

The first of these questions has been intentionally posed naïvely, to permit the answer to include an obvious point: it has always been clear that either distributed or sufficiently fine-grained, lumped compensation can yield satisfactory results, as far as accelerator theory is concerned. Deciding between them depends on administrative weighting of various factors: manufacturing feasibility, desirability of separating functions, preserving flexibility, cost, operational ease, and so on. Most of these factors are outside the province of this working group. In particular, regarding operational issues, a correction scheme that is too sophisticated or requires beam or magnet measurements that are impractical will be risky from the point of view of reliable machine operation. Whether a particular correction scheme might be operationally impractical has not been addressed in these studies, except in the "gut reactions" of the members of the working group. It should be kept in mind that other, more direct methods of increasing the aperture, such as increasing the injection energy or enlarging the coil diameter, could be preferred from these, broader considerations, although no technical results in this study has made this consideration compulsory. An assignment for the working group has been to generate input for a broader comparison by zeroing in on one or more simple, economical, and practical lumped schemes, that yield performance comparably good to that achievable with the CDR distributed scheme. Two lumped schemes have resulted from this screening, each having two lumped elements per half-cell. Neither scheme yields performance superior to the distributed scheme, but the differences are small; small enough to permit us to claim to have completed that assignment successfully.

Systematic multipole errors have a large effect upon global properties like chromaticity. It has been found, in the absence of other effects, that compensation has been straightforward, even using "remote" compensation schemes having correctors many half-cells away from the errors being compensated. It is found, however, that performance of such remote schemes is degraded by the simultaneous inclusion of other errors, notably random multipoles and closed-orbit errors. One form of remote scheme, common in existing accelerators, called "harmonic compensation", has not been investigated. It perhaps should be, but the results of the present study suggest that such schemes will be similarly degraded by other errors.

Within the guidelines of the CDR, compensation of random errors has also been found to be satisfactory, with compensation of just b_2 reducing the "smear" to about 5% within the "needed aperture". Nothing in this study bears on the question of what constitutes a tolerable level of smear.

To the extent comparisons have been made, projections of the CDR have been largely born out. Examples are closed-orbit, tune, and chromaticity adjustment as anticipated there. In particular, the correction with b_2, b_3, and b_4 coils mounted on the bore-tube has permitted the compensation of both systematic and randoms as well as any other scheme studied. The remote, lumped elements present in that design have been used for successful remote compensation, but the same reservation made previously about remote compensation suggests replacing the remote elements of the CDR design. In principle, even random errors could be compensated to some degree by those remote correctors, but no practical way of doing this has been found (nor really looked for seriously).

One important issue identified in the study has been the interplay of different errors which complicates the task of compensation. This complication makes itself progressively more important as the simulation includes more effects. Most noticeable so far have been difficulty in decoupling, increases in smear, and deterioration of remote compensation schemes when closed-orbit errors are included realistically.

As well as projecting ultimate performance it is important to investigate the operational practicality of diagnostic and adjustment schemes. According to the simulation, compensation of small-amplitude behavior (mainly as a function of momentum) has been shown to be quite feasible, but large amplitude behavior has not yet been adequately investigated.

It would be valuable to quantify the "safety factor" in the SSC design, but that is very difficult and the present study has concentrated more on relative performance than on absolute performance. Certainly it has been found to be necessary to compensate more field errors than has been true of existing accelerators. With modern diagnostic data collection and processing there exists a somewhat enhanced capability of performing such tasks compared to that available when existing machines were built. Considerable progress has been made in developing, practising with, and assessing this enhanced capability but much remains to be done.

As has been stressed repeatedly, this report has been limited to a fraction of the considerations needed to motivate important changes in the CDR. Estimates have been given of the increased safety margin which could be "purchased" by increasing the injection energy and/or by increasing the bore radius of the dipoles. The latter would help primarily by improving the convergence of the multipole series, thereby reducing the importance of higher multipoles and reducing both the number of compensation elements and their criticality. To illustrate this reduced multipole sensitivity, we project that an improvement factor of 2.1 in the "worst" higher multipole effect found in our study would accompany a 25% enlargement in bore radius.

Important issues that have been left out include quadrupole errors, effects of intersection regions (even if ideal, they complicate the optics and orbit control and double the chromaticity; when not ideal, errors in those regions tend to be magnified at least for colliding beam optics), and field errors that are neither completely random nor completely systematic (due, for example to temperature variations around the ring, or to gradual drifts with time in the manufacturing processes, and drifts and randomness in persistent currents).

Finally we turn to the question of what should be studied next. Certainly it does not follow that including all the things mentioned in the previous paragraph has the highest priority. It remains true that our limited resources should be focused on the regular arcs, until they are frozen, since they dominate the SSC cost. Clearly the IR design is important, especially in maximizing the luminosity, but these concluding comments will be restricted to the regular arcs.

In this study the number of competing correction schemes has been much reduced; probably enough so that the decision whether to use lumped or distributed correction can be deferred without making excessive the analysis burden of carrying forward more than one option. It is however important to make the designs more detailed and realistic than has been true in this report; not so much because the details are important this early, but because it is the best way to motivate thorough studies.

The operational simulation capability should continue to be enhanced. It is appropriate for this capability to be developed sufficiently that it can be applied to all operational questions. The current operations simulation can be applied to the problem of decoupling of the lumped schemes with closed orbit errors. It has already been invaluable in correcting the orbit in the studies of sensitivity to closed orbit errors, in studies of two families of correctors in the spoolpieces, and, earlier, in studying the operational steps necessary to correct the chromaticity given imperfect settings of the systematic correctors.[9] The latter studies must be repeated for schemes which pass all other tests.

It is rather peripheral to this report but it should also be stated that this capability should be exercised on existing accelerators, as it was to some extent during the Fermilab experiment E778. That experiment is continuing, and experimentation on diagnostic schemes is one competitor for the accelerator time available; no committment has been made.

It seems sensible to concentrate on the question of closed-orbit sensitivities in the near future. At the present time the greatest uncertainties appear there, although studies so far have not yielded any surprises. This is largely because the detailed studies have not yet all been completed and it seems appropriate to give that the highest priority. This should include analytical work to confirm, contradict, or correct the numerical studies. Efforts should be made to apply to this problem the analytical mapping technique presently being developed. The current simulations should be expanded to include off-momentum effects. Decoupling of the lumped schemes which had large coupling coefficients with closed orbit errors should be investigated. Design changes such as having more beam position correctors and monitors could also be considered, since those can reduce the r.m.s. orbit deviations, and thereby the size of the effects of feeddown.

APPENDIX A

Effects of Systematic Nonlinear Multipoles.

Suppose that a particular nonlinear dipole multipole, $b_n^{(D)}$, differs systematically from zero. That is, summing over all N magnets in the ring,

$$\frac{1}{N}\sum_{i=1}^{N} b_{ni}^{(D)} = b_n^{(D)} \neq 0; \quad n \geq 2 \tag{A1}$$

(Systematic skew elements, a_n, will also have to be compensated but, being much smaller, they will be temporarily ignored.) Since b_2 is the largest and most important multipole, it will be treated individually and discussed explicitly in this section. Some higher order multipoles (in particular b_3, b_4, and b_6) are large enough to demand inclusion; they will be discussed, as a group, later.

It is assumed that $b_2^{(D)}$ is constant along the full length of the dipole. That is, $b_2^{(D)}(s) = b_2^{(D)}$ = constant, where s is arc length measured from the half-cell center. To simplify the discussion it will also be assumed that the quadrupoles have no length and that the dipole completely fills each half cell. Of course, in actual calculations the correct lattice dimensions are used.

Correction elements are described by the function $b_2^{(C)}(s)$. For zero-length elements these will include δ-functions. This function can be chosen to reduce various effects to acceptable levels. Some of those effects will now be enumerated, more-or-less in order of importance, with the most basic first.

(i) A particle passing through the half-cell will have accumulated, at the output, a horizontal angular deflection error given (to a lowest but thoroughly adequate approximation, and dropping a constant numerical factor) by $\Delta x'(\ell/2)$, where

$$\Delta x'(s) = \int_{-\ell/2}^{s} [b_2^{(D)} + b_2^{(C)}(s)][(x_o + x_o's)^2 - (y_o + y_o's)^2]ds \tag{A2}$$

where x_o and x_o' are horizontal displacement and slope at the magnet entry point, and similarly for y. This neglects focusing in the dipole, treats the dipole as a drift, with the straight-line, unperturbed orbit used to calculate the angular error.

(ii) The corresponding vertical output deflection error is given by

$$\Delta y'(\ell/2) = 2 \int_{-\ell/2}^{\ell/2} [b_2^{(D)} + b_2^{(C)}(s)](x_o + x_o's)(y_o + y_o's)ds. \tag{A3}$$

(iii) There will also be displacement errors given by

$$\Delta x(\ell/2) = \int_{-\ell/2}^{\ell/2} \Delta x'(s)ds \tag{A4}$$

$$\Delta y(\ell/2) = \int_{-\ell/2}^{\ell/2} \Delta y'(s)ds \tag{A5}$$

Note that the four terms described so far are specific to the individual dipole magnet, and do not depend at all on any lattice functions at the position in the lattice that the dipole happens to be located. Compensating all these terms to zero would be equivalent to making the magnet perfect; all other formulas would then be superfluous. In that sense these are the most important terms, but it is unfortunately not practical to adjust them all to zero.

(iv) The displacement error, $\Delta x(\ell/2)$, and the deflection error, $\Delta x'(\ell/2)$, can be combined to form an error of the Courant-Snyder[27] invariants:

$$\frac{\Delta \epsilon_x}{\epsilon_x} = \frac{2x\Delta x + 2x'\Delta x'\beta_x^2}{x^2 + x'^2\beta_x^2}. \tag{A6}$$

There is a corresponding formula for y. Unlike the previous quantities, these errors do depend on the lattice β-functions. When Eq.(A 6) is applied differentially to a thin element of the dipole, in lowest order, it is only the $\Delta x'$ term which contributes to the variation of the Courant-Snyder invariant. The resulting expression is approximately proportional to $\Delta x'$ as given by Eq.(A 2); the same correctors which suppress the slope error will suppress the Courant-Snyder invariant error.

(v) The global quantities which respond most sensitively to systematic magnet multipoles, and are most deleterious to operations, are the horizontal and vertical chromaticities:

$$\frac{d\nu_{x,y}}{d\epsilon} = \frac{N}{2\pi} \int_{-\ell/2}^{\ell/2} [b_2^{(D)} + b_2^{(C)}(s)]\beta_{x,y}(s)\eta(s)ds \tag{A7}$$

where, within a dipole, and again neglecting dipole focusing, the lattice functions vary as

$$\beta(s) = \beta_o - 2\alpha_o s + \gamma_o s^2; \quad \eta(s) = \eta_o + \eta_o' s + \pi/(N\ell)s^2. \tag{A8}$$

(vi) The leading nonlinear resonance driving-amplitude is proportional to

$$\int_{-\ell/2}^{\ell/2} [b_2^{(D)} + b_2^{(C)}(s)]\beta_x^{3/2} \cos(3\psi_x(s))ds \tag{A9}$$

where $\psi_x(s)$ is the betatron phase. This has only been written for reference; the factor $\cos(3\psi_x(s))$ varies so quickly over the half-cell that the granularity of compensation is usually too coarse for accurate **local** compensation. (This cloud has a silver lining: the same factor gives a strong **global** cancelling effect.)

APPENDIX B

Compensation of Systematic Nonlinear Multipoles.

Of the effects analysed in Appendix A, all but the last (and least important) were described by formulas consisting of sums of terms of the form

$$\int_{-\ell/2}^{\ell/2} [b_2^{(D)} + b_2^{(C)}(s)] s^p ds \tag{B1}$$

where p is a small integer. A natural approach to compensation is to choose $b_2^{(C)}(s)$ to make these terms small for values of p not greater than some value p_{max}. That is

$$\int_{-\ell/2}^{\ell/2} b_2^{(C)}(s) s^p ds \simeq I_p \tag{B2}$$

where

$$\begin{aligned} I_p =& - b_2^{(D)} \frac{2(\ell/2)^{p+1}}{p+1}, \quad \text{p even,} \\ =&0, \qquad\qquad\qquad\qquad \text{p odd.} \end{aligned} \tag{B3}$$

Fully-lumped schemes. The entire compensation can be performed using N_L short lumped correctors, of lengths L_i, located at $s = s_i$, for $i = 1, 2, ..., N_L$. Eq. (B 2) becomes

$$\sum_{i=1}^{N_L} L_1 b_{2i}^{(C)}(s_i) s_i^p = I_p, \quad p = 0, 1, ..., p_{max}. \tag{B4}$$

As emphasized first by Neuffer,[4] these conditions are equivalent to numerical quadrature formulas. The two most promising candidate formulas are:

(*i*) Simpson's rule.

$$\int_{-1}^{1} f(s)ds = \frac{1}{3}[f(-1) + 4f(0) + f(1)] - 0.011 f^{iv}(\xi) \qquad \text{(B5)}$$

where $f^{iv}(\xi)$ is a fourth derivative evaluated somewhere in the range. It has usually been assumed, and this will be investigated further as part of the study, that the lumps on either side of a lattice quad (represented by the terms $f(-1)$ and $f(1)$) in (B 5) can be combined into a single lump. This is now indicated pictorially, labelling correctors F, D, or C, depending on whether they are beside F or D quads, or in the center:

$$\Big|\Big)\, F_+[\ ][\ ][\ ]C[\ ][\ ][\ ]D_- \Big)\Big(\, D_+[\ ][\ ][\ ]C[\ ][\ ][\ ]F_- \Big(\Big|;$$

The correctors, D_- and D_+, would be lumped together on one or the other side of the D-quad, and the F-correctors would be similarly lumped. The result is shown:

$$\Big|\Big)\, F[\ ][\ ][\ ]C[\ ][\ ][\ ] \Big)\Big(\, D[\ ][\ ][\ ]C[\ ][\ ][\ ] \Big(\Big|;$$

(*ii*) Gaussian quadrature.

$$\int_{-1}^{1} f(s)ds = [f(-0.58) + f(0.58)] + 0.0074 f^{iv}(\xi) \qquad \text{(B6)}$$

where $f^{iv}(\xi)$ has the same significance as in Simpson's rule. The excellent properties of this scheme were pointed out by Forest and Neuffer.[28] The reason it is somewhat more accurate than the Simpson formula, in spite of having fewer lumps (2 instead of 3), is that the abscissas ± 0.58 are optimal. (In this connection, note also the comment above about combining end-correctors in the Simpson-Neuffer scheme.) Unfortunately, with 6 dipoles per half-cell, as in the present SSC design, those points are not accessible. With either 5 or 10 dipoles per half cell the points ± 0.60 are accessible, one fifth of the way along the half-cell; it is expected that that is adequately close. Pictorially, indicating the lumped correctors by G:

$$\Bigg|\Bigg)\,[\ \ ]G[\ \ ][\ \ ][\ \ ]G[\ \ ]\Bigg)\Bigg(\,[\ \ ]G[\ \ ][\ \ ][\ \ ]G[\ \ ]\,\Bigg(\Bigg|$$

(It is known to be too constraining for all G-correctors to have the same strength, but the extra flexibility can be achieved remotely.)

APPENDIX C

Sensitivity of the Chromaticity to Non-Optimal Corrector Location and Symmetry.

We first estimate the chromaticity due to a given systematic sextupole coefficient b_2. An ideal compensating coil would have strength $-b_2$, (referred to the dipole field of the dipole on which it is exactly superimposed,) and that would exactly cancel the effect of the original error. If the corrector is translated a distance d along the beam line from its optimal position, there will be a residual chromaticity proportional, in lowest order, to d. A formula will be derived and applied to the SSC. First define symbols:

$$
\begin{aligned}
n &= \text{total number of half cells } (= 640) \\
n_D &= \text{number of magnets per half cell } (= 6) \\
\Delta\theta_1 &= \text{bend in one magnet} = 2\pi/(nn_D) \\
\Delta x' &= \text{angular deflection due to sextupole} \\
&= b_{2i} 2\pi (x_\beta + \epsilon\eta_i)^2/(nn_D)
\end{aligned}
$$

where the particle has betatron amplitude x_β and fractional energy offset ϵ, at a point in the lattice where the dispersion is η_i. Extracting the coefficient of the term proportional to x_β, which is the effective quadrupole strength of the error field, yields $b_{2i}4\pi\epsilon\eta_i/(nn_D)$. Inserting this in a standard formula yields the tune shift $\Delta\nu$ due to this quadrupole; that in turn yields the chromaticity

$$
\frac{\Delta\nu}{\epsilon} = \frac{1}{n_D}\sum_{i=1}^{n_D} b_{2i}\beta_i\eta_i, \tag{C1}
$$

where the sum has been reduced to a sum over just the magnets in a single half-cell. Alternatively, this could be expressed as an integral

$$
\begin{aligned}
\frac{\Delta\nu}{\epsilon} &= \frac{1}{\ell}\int_0^\ell b(s)\beta(s)\eta(s)ds \\
&\simeq <b_2><\beta>_g<\eta>_a
\end{aligned} \tag{C2}
$$

$$\text{where} < \beta >_g = \sqrt{(\beta_{min}\beta_{max})} = \ell / \sin\phi$$
$$\text{and} < \eta >_a = 0.5(\eta_{min} + \eta_{max}) \simeq < \beta >_g / \nu \tag{C3}$$

are average values of the lattice functions; ℓ is the length of the half-cell, ϕ is the phase advance per half-cell. Using SSC values, $< b_2 >= 7.4 \times 10^{-4}$, $< \beta >_g = 141m$, and $< \eta >_a = 1.7m$, yields 1774 as the value of the uncompensated chromaticity. (Taking $\epsilon = 10^{-3}$ and dividing by 0.001 to express this –as elsewhere in this report– as a tune shift in units of 0.001 leaves this number unchanged.)

This is the gigantic chromaticity to be compensated. It should not be surprising that slightly non-optimal compensation can leave a still-unacceptable chromaticity, as we now estimate. Suppose all b_2 compensation coils have the correct strength, $b_2 = -7.4$units, but are systematically displaced along the beam line by a distance d from where they should be. The integrand of the integral by which the chromaticity is calculated will vanish over much of the range, but there will be a large contribution $b_2\beta_{max}\eta_{max}d/\ell$ from one end, and a much smaller term from the other end which will be ignored. In the current SSC design the b_2 bore-tube correctors are at one end of each magnet and have approximately half of the magnet length. Effectively then, they are displaced by $d = 4m$, from their ideal locations. Taking $\beta_{max} = 324m$, $\eta_{max} = 4.0m$, $\ell = 100m$, yields an error of 384; that is, the error is only reduced to 384/1774=22% of its uncorrected value. This accounts for the large effects observed in Section 5.1.

If the coils are displaced in the same direction in the next cell, as in BORCDR for example, then this error is largely compensated. This is the least extreme example of remote compensation, in which imperfection in one part of the lattice is compensated in another. Compensation such as this brings with it the risk of depending on features present in the ideal lattice which, owing to other sources of error, are not present in the actual lattice. It is explained in Appendix B that the local cancellation of certain error integrals (odd moments of s) depends on the corrections being symmetrically placed relative to half-cell centers. In the case of BORCDR this is not satisfied; that accounts for the large half-cell chromaticity. (This has not been studied extensively, but so far cancellation on the full-cell basis has been found to be sufficiently local to protect against global imperfection.) The elegançe of Gaussian compensation schemes, of which GAUI, having two lumps per half-cell, is the simplest, is that they simultaneously cancel odd moments of s (by symmetry) and even moments (by placement,) thereby achieving truly local, lattice insensitive, compensation.

APPENDIX D

(a) Expected Field Errors and Their Reliability.

The multipole errors used in this corrector study are those due to geometric factors and to persistent current effects. Yoke saturation reduces the normal sextupole coefficient b_2 by something in the range of 0 to -2 units, depending on the design of the iron yoke, but since this change is small relative to that due to persistent currents it is not specifically included in this study. Saturation effects on the other multipoles have been measured to be much smaller (0.1 unit or less).

The geometric multipoles for the SSC dipole magnets are taken from the specifications listed by Chao and Tigner,[18] , modified slightly to reflect the change to the 90-degree lattice.[29]

These multipoles are listed in Table D.1. The persistent current multipoles in the dipole magnets at the injection field of 0.331 T are based on calculation by Green[30] and are also listed in Table D.1. The total systematic multipole coefficients used in the present calculations are a judicious combination of the geometric, systematic and the persistent-current coefficients and are separately listed in Table D.1.

The reliability of these geometric multipole projections can be judged to some extent by comparing with the measured coefficients[31] of the various SSC model dipole magnets, which also are listed in Table D.1. However, in making such comparisons, one must keep in mind that in the model magnets several coil cross sections are represented and probably do not include the final cross section, so that the measured geometric systematic and r.m.s. variations of the allowed coefficients (b_2, b_4, b_6, b_8, . . .) are probably not representative of the final dipole magnet design. Also listed in Table D.1 are the average (and r.m.s. variations) of the allowed persistent current multipoles[32] measured at 0.33 T in five 1.8-m (BNL) and eight 1-m (LBL) model magnets made with 5μ filaments and scaled to coils with 6μ filaments with $J_c = 2750$ A/mm^2.

Table D.1 Magnetic Multipole Coefficients Used in the CEWG Study. Units are 10^4 B_0 at 1 cm.								
	$a_1^\dagger$	a_2	a_3	a_4	a_5	a_6	a_7	a_8
systematic spec.	0.2	0.1	0.2	0.2	—	—	—	—
r.m.s. variation spec.	± 0.7	± 0.6	± 0.7	± 0.2	±0.2	± 0.1	± 0.2	± 0.1
7 long models, syst.	0.55	0.3	0.09	0.02	0.02	–0.01	0.01	–0.01
7 long models, r.m.s. var.	± 0.76	± 0.25	± 0.16	± 0.06	± 0.04	± 0.02	± 0.02	± 0.02
6 1.8-m models,* syst.	0.00	0.08	–0.09	0.02	0.01	0.01	0.01	0.01
6 1.8-m models, r.m.s. var.	± 1.1	± 0.40	± 0.43	± 0.10	± 0.06	± 0.02	± 0.02	± 0.01
6 1.0-m models, syst.	0.08	0.62	0.08	0.08	–0.01	–0.02	0.00	0.01
6 1.0-m models, r.m.s. var.	± 1.8	± 0.66	± 0.36	± 0.04	± 0.08	± 0.02	± 0.01	± 0.01
	$b_1^\dagger$	b_2	b_3	b_4	b_5	b_6	b_7	b_8
systematic spec.	0.2	1.0	0.1	0.2	0.04	0.07	0.1	0.2
r.m.s. variation spec.	± 0.7	± 2.0	± 0.3	± 0.7	± 0.1	± 0.2	± 0.2	± 0.1
persistent-current calc.[&]	—	-7.9	—	0.75	—	–0.14	—	—
total systematic used	0	-7.4	0.1	0.64	0	-0.13	0	0
7 long models,[#] syst.	0.76	—	0.04	—	0.00	—	0.00	—
7 long models, r.m.s. var.	± 1.6	—	± 0.07	—	± 0.02	—	± 0.01	—
6-10 1.8-m models,*syst.	–0.11	1.05	–0.02	0.27	–0.01	–0.08	–0.01	0.03
6-10 1.8-m models, r.m.s. var.	± 1.1	± 1.8	± 0.17	± 0.43	± 0.07	± 0.09	± 0.02	± 0.05
6 1.0-m models, syst.	–0.03	3.3	–0.10	0.02	–0.04	0.09	0.01	0.00
6 1.0-m models, r.m.s. var.	± 1.5	± 1.9	± 0.13	± 0.93	± 0.07	± 0.10	± 0.01	± 0.02
5 1-m & 8 1.8-m models	—	–7.6	—	0.79	—	–0.23	—	—
persistent-current meas.	—	± 1.8	—	± 0.33	—	± 0.14	—	—

† In the CEWG study, systematic and random a_1 and b_1 were set to zero.

& M. A. Green, private communication, May 88, scaled to 6μ filament, $J_c = 2750$ A/mm^2

at 5T, 4.2K. Lower values were used early in the CEWG study.

\# The allowed normal multipoles were not averaged because the 7 long magnets included three different coil designs.

* DSS9 data excluded (NC9 cross section).

(b). Multipole Values Appropriate for Doubled Injection Energy

and for Bore Diameter Enlargement From 4cm to 5cm.

Table D.2 Table of Multipoles for CEWG Consideration								
Normal Multipoles								
	Systematic						Random	
	Geometric		Persistent Current					
			1 Tev Inj.		2 Tev			
Multipole	4cm	5cm	4cm	5cm	4cm	5cm	4cm	5cm
b_1	0.2	0.15	—	—	—	—	0.7	0.56
b_2	1.0	0.63	– 7.4	– 4.7	– 3.0	– 1.9	2.0	1.36
b_3	0.1	0.054	—	—	—	—	0.3	0.18
b_4	0.2	0.092	0.64	0.30	0.20	0.09	0.7	0.35
b_5	0.04	0.016	—	—	—	—	0.1	0.043
b_6	0.07	0.024	– 0.13	– 0.044	– 0.050	– 0.017	0.2	0.073
b_7	0.1	0.029	—	—	—	—	0.2	0.063
b_8	0.2	0.050	—	—	—	—	0.1	0.027
Skew Multipoles								
a_1	0.2	0.15					0.7	0.56
a_2	0.1	0.06					0.6	0.41
a_3	0.2	0.11					0.7	0.41
a_4	0.2	0.09					0.2	0.10
a_5	—	—					0.2	0.09
a_6	—	—					0.1	0.037
a_7	—	—					0.2	0.063
a_8	—	—					0.1	0.027

Notes:

(1) Systematic-geometric scaled as $(R_c)^{n+1}$ for constant errors.

(2) Systematic-persistent scaled as $(R_c)^{n+1}$ for constant filament diameter.

(3) Random multipoles scaled as $(R_c)^{n+\frac{1}{2}}$ where R_c is the effective coil radius, taken as 3cm for 4cm inner diameter (ID) dipole and 3.5cm for the 5cm ID dipole.

(4) Energy dependence of persistent-current multipoles taken from theoretical calculations by M.A. Green.[30]

APPENDIX E

Effects of non-uniform Cells.

The standard 90° lattice is composed of uniform cells with one large gap of 6.57 m for a spoolpiece on the clockwise side of the quadrupole in each half cell, with a 0.8 m inter-magnet gap everywhere else. These gaps are between magnetic effective lengths. This appendix considers the consequences of introducing an additional gap in the center of each half cell of a cell, once every N^{th} cell.

Two different cases were studied – one in which every 10^{th} cell had a gap of 4.1 m between the magnetic ends of dipoles 3 and 4 in each of its half cells, and another which had similar gaps every 9^{th} cell. The effects on the linear optics of these perturbations were measured by examining the beta and dispersion-function distortions over the unperturbed case. The results are that the N = 10 cell case is completely unsatisfactory, due to the 10-cell tune, while the N = 9 cell case is tolerable in terms of its linear optics properties. Plots of the beta and eta functions for the perturbed 9 and 10 cell cases are shown below, and Tables E.1 and E.2 give the beta and eta distortions for the two cases for a variety for cell tunes. The current SSC design calls for a maximum tuning range of ±1 unit of tune per arc, corresponding to a maximum beta distortion of 6% for N = 9. The dispersion distortion is similar. These distortions would necessitate re-matching the optics of the straight sections, but would not seriously complicate the lattice design.

Table E.1

Maximum cell lattice function values for nine regular cells and for nine regular cells plus one cell with two 4.1 meter gaps as a function of the ten-cell tune.

Structure	ν_x	ν_y	β_x	β_y	η
10 Cells	2.492	2.492	388m	388m	3.05m
9 + 1 Cell	2.485	2.492	445	912	3.12
	2.476	2.476	420	522	3.13
	2.439	2.439	402	441	3.20
	2.349	2.349	396	401	3.38

Table E.2

Maximum cell lattice function values for nine regular cells and for eight regular cells plus one cell with two 4.1 meter gaps as a function of the nine-cell tune.

Structure	ΔQ/arc	ν_x	ν_y	β_x	β_y	η
9 Cells	0	2.243	2.243	388m	388m	3.05m
8 + 1 Cell	1.9	2.362	2.362	420	427	2.91
	1.0	2.301	2.301	409	412	3.03
		2.243	2.243	403	402	3.14
	-1.0	2.181	2.181	401	393	3.28
	-1.9	2.124	2.124	402	395	3.44

Figure E.1. Nine regular cells and one cell with two 4.1 meter gaps.

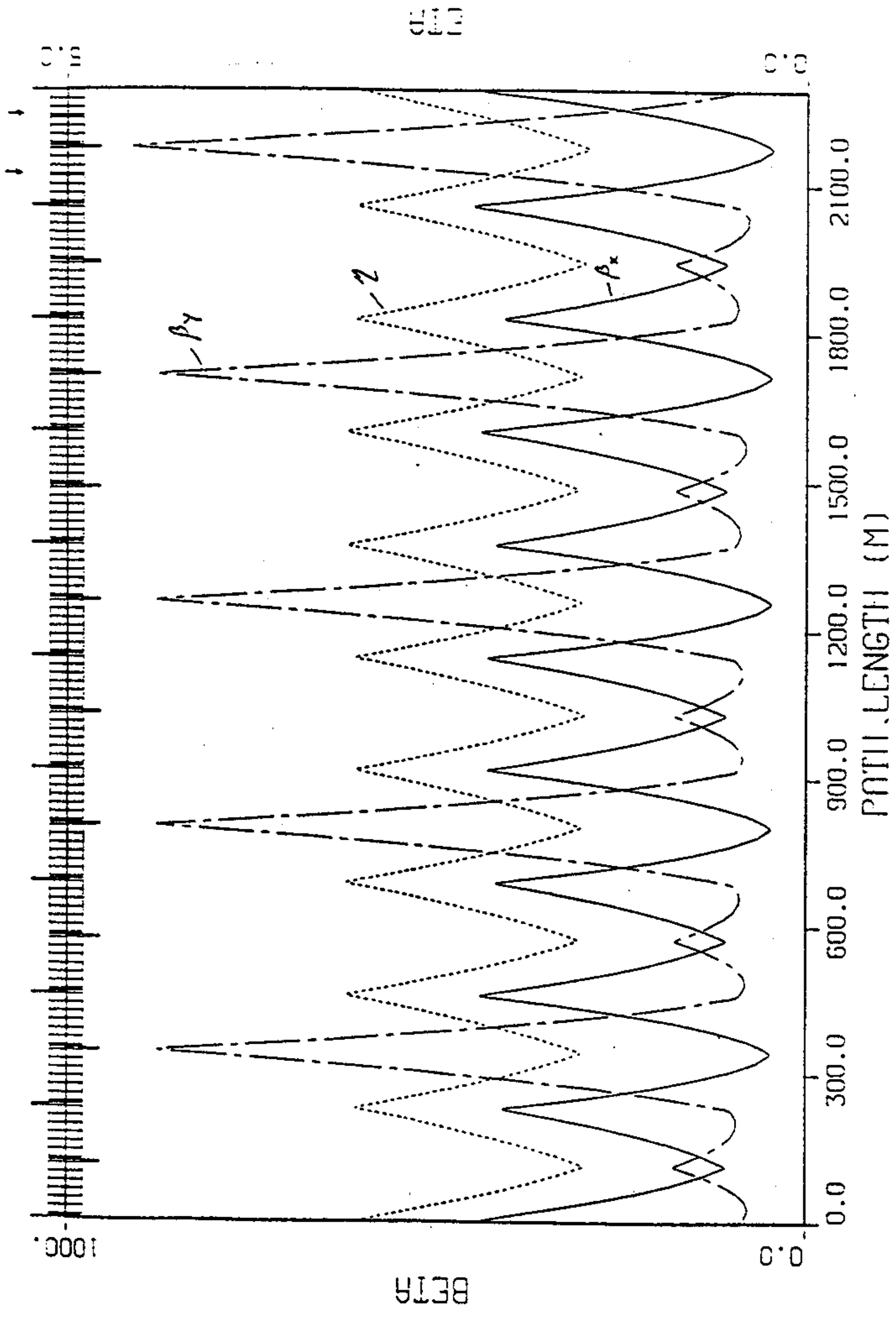
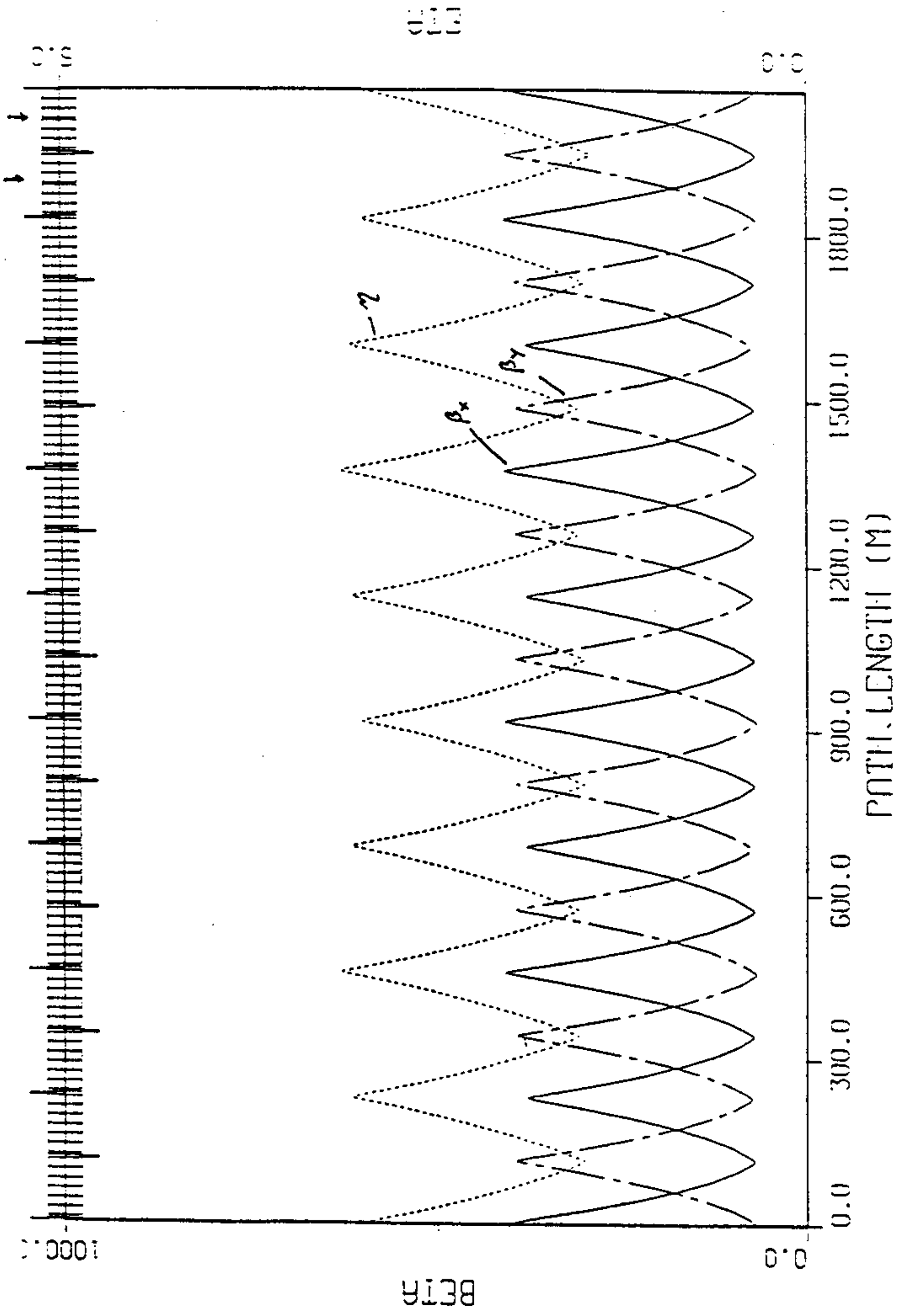


Figure E.2. Eight regular cells and one cell with two 4.1 meter gaps.



APPENDIX F

Numerology for the Remote Placement of Systematic Correctors.

The b_n correctors in a remote correction scheme, b_3, b_4 and b_6, are placed in a configuration which repeats itself every N cells, where N is chosen so as to avoid resonance build-up from the correctors. It is assumed that N is the same for n = 3, 4 and 6, even though all three correctors may not be combined into a single hardware package - and the three configurations may even be different from one another. What is a good value for N?

If the same pattern of b_n correctors is repeated every N cells, then one of several terms which distorts transverse phase space dynamics is proportional to

$$S_{nkl} = b_n \sum_j \exp[ijN(k\delta\phi_H + l\delta\phi_V)] \tag{F1}$$

where

$$\begin{aligned} i &= \sqrt{-1} \\ j &= 0, 1, 2 ... \\ k &= n+1, n+1-2, ...(n+1)\text{mod}2 \\ l &= \text{integer}(\frac{n+1}{2}), ...2, 0, -2, ... - \text{integer}(\frac{n+1}{2}) \\ & \text{with} \quad k + |l| \leq n+1 \end{aligned} \tag{F2}$$

In the case at hand the phase advances per cell, $\delta\phi_H$ and $\delta\phi_V$, are both 90 degrees, or $\pi/2$, so that

$$S_{nm} = b_n \sum_j \exp[2\pi ij \frac{N(n+1-2m)}{4}] \tag{F3}$$

where m = 0, 1, 2 ..... integer($\frac{n+1}{2}$) .

Notice that the same result would be obtained if the correctors were skew, except for changing b_n to a_n.

Resonant build up of the vector S_{nm} is avoided by ensuring, wherever possible, that the phase of the vector does not remain constant as j increases, stepping from corrector to corrector. There is no resonant build up IF

1) N mod 4 $\neq$ 0 AND

2) N mod 2 $\neq$ 0 OR (n + 1 - 2m) mod 2 $\neq$ 0 AND

3) (n + 1 - 2m) mod 4 $\neq$ 0

If n is even (n mod 2 = 0), then conditions 2) and 3) are automatically satisfied, leaving only the condition that N not be divisible by 4.

If n is odd (n mod 2 = 1), then conditions 1) and 2) are only satisfied if N is odd. Even so, condition 3) is only satisfied for every other value of m . For example, when n = 3, the terms with m = 0 and m = 2 can not be suppressed.

The conclusion so far, then, is simply that the best choice for N is ANY ODD NUMBER, a choice which suppresses all distortion terms for even-n correctors, but which only suppresses half of the odd-n corrector terms.

The following argument shows that this conclusion is not modified when longitudinal phase space is included. A test particle with an energy offset of $\delta = \delta E/E$ and a horizontal offset of x passing through a corrector with a dispersion function of η receives a horizontal angular kick of

$$\Delta x' = kb_n(x + \eta\delta)^n = kb_n(x^n + n(\eta\delta)x^{n-1} + ... + (\eta\delta)^n) \qquad \text{(F4)}$$

Here k is a constant of proportionality. The additional terms in the binomial expansion, such as $n(\eta\delta)x^{n-1}$, show that b_n correctors masquerade as lower order correctors, such as $b_n - 1$, in the presence of energy errors. This effect is often referred to as the "feed down" phenomenon. The relative strength of the feed down terms is fixed by the magnitude of $\eta\delta$, that is, by the magnitude of the energy offset. As a crude rule of thumb, the feed down terms are all commensurate for large values of δ in the SSC. Regardless of whether n is even or odd, then, a set of b_n correctors will excite both even and odd order distortions, one quarter of which will receive no suppression from the addition of vectors with different phases. The best strategy is still to choose N odd, however.

Finally, consider the effect on the linear lattice of a repeating pattern of additional "drift" lengths every N'th cell for the placement of systematic remote correctors. In their effect on betatron functions, these drifts act like b_2 correctors in the formalism developed above, while they have a b_1 effect on the dispersion function. It is apparent that if N divides by 2, then erroneous betatron waves are coherently excited, and that if N divides by 4, then dispersion waves are coherently excited. If N is odd, however, then both kinds of error wave are suppressed.

APPENDIX G

A Naive Estimate of Smear Due to Displaced Lumped Correctors.

The perturbation in the horizontal amplitude a measured at a reference point with β_0 due to a nonlinear magnet is

$$\delta a = a_c \sum_{n=2}^{\infty} b_n x^n \tag{G1}$$

where

$$a_c = \sqrt{\beta_0 \beta_c} \frac{B_0 L_c}{B\rho} \tag{G2}$$

Here β_c and L_c are the beta function and length of the magnet, and the b_n follow the conventional TEAPOT definition. On one particular turn, the displacement of a particle of amplitude a at the magnet is

$$x = a\sqrt{\beta_c/\beta_0} \sin \Psi \tag{G3}$$

Following the spirit of SSC-20, the smear is estimated from the total perturbation due to N such random magnets, encountered in one turn. Very loosely, then, the smear S is

$$\begin{aligned} S &\approx 2 \frac{< \delta a^2 >^{\frac{1}{2}}}{a} \\ &= 2\sqrt{N} a_c \sum_{n=2}^{\infty} C_n \sigma_{b_n} a^{n-1} \\ \text{where} \quad C_n &= \left(\frac{\beta_c}{\beta_0}\right)^{n/2} < \sin^{2n} \Psi >^{\frac{1}{2}} \\ &= \left(\frac{\beta_c}{\beta_0}\right)^{n/2} \frac{((2n)!)^{\frac{1}{2}}}{2^n n!} \end{aligned} \tag{G4}$$

and σ_{b_n} is the standard deviation of b_n. This estimate inevitably handles resonance effects incorrectly, and amounts to little more than dimensional analysis.

Now consider the magnets to be lumped correctors, with only one systematically set non-zero value of b_n, so that the σ_{b_n} are normally all zero. Non-zero values of σ_{b_n} are due to first order feed down through random displacements, d, of the correctors. Then it is reasonable to replace

$$\Delta x' \sim b_n(x-d)^n \quad \text{with} \quad \Delta x' \sim b_n(x^n - ndx^{n-1}) \tag{G5}$$

(This seems appropriate for d $\ll$ a, and is presumably simple to prove, but is unconfirmed.) Thus,

$$\sigma_{b_{n-1}} = n \mid b_n \mid \sigma_d \tag{G6}$$

where σ_d is the standard deviation of the displacements. Note that if these displacements are to be considered as due to closed orbit errors, then there should be an extra factor of $\sqrt{\beta_c/\beta_0}$ included here. So, the approximate smear due to one such set of correctors is expected to be

$$S \approx 2\sqrt{N}\sigma_d.a_c.n \mid b_n \mid C_{n-1}(0.005)^{n-2}.\left(\frac{a}{0.005}\right)^{n-2} \tag{G7}$$

The periods in this expression separate four terms of different character. The first term is constant over all corrector sets, except, perhaps, for a factor of $\sqrt{2}$ or so. The second term, a_c, depends only on the corrector set location. The third term depends on the corrector set and on the n value of the multipole, while the fourth term has been pre-scaled to a typical amplitude of 5 millimeters.

For example, in BFUL5C there are F, D, and M (for middle) correctors in two half cells, every fifth cell. If the half cell length is 115 metres, and the cell phase angle is 90 degrees, then β_F ($=\beta_0$), β_M, and β_D are 393, 173 and 67 metres, respectively. Take σ_d = 1 millimeter, and ($B_0L/B\rho$) = 0.0147, 0.0589, 0.0147 for F, M, and D corrector sets, with N = 128 for 640 half cells in a complete SSC. Using $b_3 = -10.0m^{-3}$ and $b_4 = -6.4 \cdot 10^3 m^{-4}$, then the contributions to the smear at an amplitude of 5 millimeters are estimated to be

n	corrector type	5mm smear, %
3	F	1.20
3	M	1.40
3	D	.08
4	F	4.68
4	M	3.65
4	D	.14

These partial smears, when added in quadrature, are in reasonable agreement with the total smear found from simulation.

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